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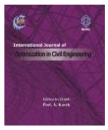

PARAMETER-FREE STRUCTURAL OPTIMIZATION OF DOME TRUSSES: DEVELOPMENT AND APPLICATION OF THE SA_EVPS ALGORITHM WITH STATISTICAL LEARNING MECHANISMS

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ABSTRACT

This study presents the application of the Self-Adaptive Enhanced Vibrating Particle System (SA-EVPS) algorithm for large-scale dome truss optimization under frequency constraints. SA-EVPS incorporates self-adaptive parameter control, memory-based learning mechanisms, and statistical regeneration strategies to overcome limitations of traditional metaheuristic algorithms in structural optimization. The algorithm's performance is evaluated on three benchmark dome structures: (1) a 600-bar single-layer dome with 25 design variable groups, (2) an 1180-bar single-layer dome with 59 design variable groups, and (3) a 1410-bar double-layer dome with 47 design variable groups, all subject to natural frequency constraints. Comparative analysis against five state-of-the-art algorithms—Dynamic Particle Swarm Optimization (DPSO), Colliding Bodies Optimization (CBO), Enhanced Colliding Bodies Optimization (ECBO), Vibrating Particles System (VPS), and Enhanced Vibrating Particles System (EVPS)—demonstrates SA-EVPS's superior convergence characteristics and solution quality. Results show that SA-EVPS consistently achieves the lowest structural weights with remarkable stability across all test cases. The algorithm's self-adaptive mechanisms eliminate manual parameter tuning while the statistical regeneration mechanism prevents premature convergence in large-scale optimization problems. This research establishes SA-EVPS as a robust and efficient metaheuristic for frequency-constrained structural optimization of complex dome structures.

Keywords: SA_EVPS algorithm; dome truss optimization; frequency constraints; metaheuristic optimization; large-scale structural design; self-adaptive algorithms.

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1. INTRODUCTION

Large-scale structural optimization presents significant computational challenges in modern civil and aerospace engineering applications. The simultaneous minimization of structural weight while satisfying complex constraints—particularly natural frequency requirements—creates highly nonlinear, non-convex optimization landscapes with multiple local optima [1]. Dome truss structures, characterized by their three-dimensional spatial configurations and efficient load distribution mechanisms, represent one of the most challenging classes of structural optimization problems due to their geometric complexity and dynamic behavior requirements.

The emergence of metaheuristic optimization algorithms has revolutionized structural design optimization over the past two decades. These bio-inspired and physics-based algorithms offer robust solutions for problems where traditional gradient-based methods fail due to discontinuous design spaces and complex constraint formulations [2]. However, the increasing scale and complexity of modern structures—with thousands of design variables and intricate constraint relationships—demand more sophisticated algorithmic approaches that can maintain both solution quality and computational efficiency.

Recent advances in metaheuristic algorithms for structural optimization have focused on hybrid approaches, self-adaptive mechanisms, and enhanced exploration-exploitation balance. Houssein et al. [3] introduced the BES-GO hybrid algorithm, combining Bald Eagle Search with Growth Optimizer techniques, demonstrating superior performance on structural design problems with standard deviations as low as 7.92E-16 in some cases. Similarly, Al Ali et al. [4] developed the Mutation-based Virus Pandemic Optimization algorithm, achieving weight reductions of 0.18%–24.2% across tested structures while requiring 61.62% fewer structural analyses than competitors.

The Vibrating Particles System family of algorithms, introduced by Kaveh and Ilchi Ghazaan [5], has shown particular promise for frequency-constrained optimization problems. The algorithm's foundation in single-degree-of-freedom vibration theory provides intuitive parameter interpretation and effective balance between exploration and exploitation. Kaveh and Talatahari [6] demonstrated the effectiveness of charged system search for geometry and topology optimization of geodesic domes, achieving optimal designs through simultaneous size, geometry, and topology optimization. Recent work by Kaveh [7] provided a comprehensive review of optimal analysis methods using the force method, highlighting the advantages of graph-theoretic approaches for achieving sparse, well-structured, and well-conditioned structural matrices essential for efficient iterative optimization.

Frequency-constrained optimization of dome structures presents unique challenges that traditional algorithms struggle to address effectively. Sarjamei et al. [8] identified key difficulties including non-convex search spaces arising from eigenvalue problems, computational expense of repeated frequency analysis, constraint handling complexity for multiple frequency bounds, and premature convergence in high-dimensional design spaces. The systematic review by Panagant et al. [9] reveals that while numerous metaheuristic algorithms exist, many exhibit poor convergence reliability for constrained truss optimization problems. Specifically, the International Student Competition in Structural Optimization benchmark studies demonstrate that even state-of-the-art algorithms often fail to converge to feasible solutions for highly constrained large-scale problems.

Despite significant algorithmic advances, several critical limitations persist in current metaheuristic approaches for large-scale dome optimization. Parameter sensitivity remains a primary barrier to practical application, as most existing algorithms require extensive parameter tuning for different problem scales and constraint configurations. Rajwar et al. [10] identified that optimal parameter values are problem-dependent and difficult to determine a priori, particularly for complex structural problems. Additionally, large-scale problems often exhibit poor convergence consistency across multiple optimization runs. The comprehensive comparative study by Khodadadi et al. [11] revealed significant performance variation even among well-established algorithms when applied to multi-variable truss problems with frequency constraints.

Traditional metaheuristic algorithms struggle to maintain proper search balance between exploration and exploitation in high-dimensional spaces typical of large-scale dome structures. Tejani et al. [12] noted that algorithms often converge prematurely to suboptimal solutions or waste computational resources on unnecessary exploration. Furthermore, frequency constraints introduce eigenvalue computations that significantly increase computational cost, and current algorithms lack sophisticated mechanisms to handle these expensive constraint evaluations efficiently while maintaining solution quality.

This research addresses these identified limitations through development and application of the Self-Adaptive Enhanced Vibrating Particle System algorithm for large-scale dome truss optimization. The primary objectives include formulating SA_EVPS with self-adaptive parameter control mechanisms that eliminate manual tuning requirements while maintaining convergence reliability across different problem scales. The research conducts comprehensive comparative analysis against five established algorithms—DPSO, CBO, ECBO, VPS, and EVPS—using three benchmark dome structures with varying complexity levels. Additionally, it evaluates algorithm performance consistency from medium-scale 600-bar to large-scale 1410-bar dome optimization problems with frequency constraints, and demonstrates SA_EVPS effectiveness for real-world structural design scenarios through detailed case studies and sensitivity analysis.

The primary contributions of this work encompass several key areas. SA_EVPS incorporates novel self-adaptive mechanisms including dynamic weight control, memory-based learning, and statistical regeneration strategies specifically designed for large-scale structural optimization problems. The systematic performance comparison across three dome configurations provides insights into algorithm scalability and constraint handling effectiveness for frequency-constrained optimization. Complete algorithmic formulation with implementation details enables adoption by structural engineering practitioners and researchers, while analysis of convergence behavior and parameter sensitivity provides theoretical insights into self-adaptive mechanism effectiveness for structural optimization applications.

2. METHODOLOGY: SA_EVPS ALGORITHM

The Self-Adaptive Enhanced Vibrating Particle System (SA_EVPS) algorithm [13-15] represents an advanced extension of the Enhanced Vibrating Particles System (EVPS) [16-18] that incorporates sophisticated self-adaptive parameter control mechanisms and statistical learning capabilities. This algorithm builds upon the physical foundation of single-degree-of-freedom vibration with viscous damping while introducing innovative adaptation strategies to enhance convergence reliability and solution quality for complex large-scale optimization problems commonly encountered in structural engineering applications.

The algorithm begins by initializing a population of N particles randomly distributed within the feasible design space, where each particle's position corresponds to a potential solution to the optimization problem. The initialization process ensures comprehensive coverage of the search space while maintaining feasibility with respect to problem constraints. SA_EVPS employs an enhanced position update mechanism that intelligently combines multiple information sources, including historically best solutions, superior solutions from the current population, and carefully selected inferior solutions that contribute to maintaining population diversity. This multi-source approach significantly enhances the algorithm's ability to balance exploration and exploitation throughout the optimization process.

A fundamental innovation in SA_EVPS is the implementation of a self-adaptive damping function that dynamically controls exploration intensity based on real-time assessment of search progress and convergence characteristics. This damping function automatically adjusts throughout the optimization process, enabling smooth transitions from global exploration in early stages to focused local exploitation as the algorithm approaches convergence. This adaptive mechanism effectively eliminates the need for manual parameter tuning, which has been identified as a significant limitation in many traditional metaheuristic algorithms. The damping function operates in conjunction with dynamic weight control mechanisms that automatically adjust the influence weights assigned to different solution components based on continuously monitored search progress and population diversity metrics.

The algorithm maintains a sophisticated memory system that archives historically best solutions to preserve diversity and prevent premature information loss. Unlike conventional approaches that typically utilize only the globally best solution, SA_EVPS implements a strategic random selection mechanism from the memory archive, which actively promotes exploration while preventing premature convergence to local optima. The memory management system includes intelligent update rules that consider both solution quality and diversity metrics, ensuring that high-quality solutions effectively guide the search process while maintaining population diversity. This memory system is complemented by a statistical regeneration mechanism where a predetermined portion of the population undergoes regeneration based on statistical characteristics of successful solutions. This mechanism leverages statistical properties derived from memory solutions to generate new candidate solutions, effectively enabling the algorithm to learn from collective swarm intelligence patterns.

The complete SA_EVPS algorithm as figure 1 follows a systematic workflow that begins with comprehensive initialization of population parameters and memory structures. The algorithm then proceeds through iterative cycles of evaluation, adaptation, and position updates until convergence criteria are satisfied. Each iteration incorporates advanced constraint handling mechanisms to ensure feasibility of generated solutions, employing adaptive penalty methods and feasibility-preserving operators that maintain constraint satisfaction throughout the optimization process. The evaluation phase computes objective function values while efficiently handling constraint violations, and the self-adaptive parameter update phase calculates population statistics and diversity measures to guide dynamic parameter adjustments.

Key improvements over the standard EVPS algorithm include the complete elimination of manual parameter tuning requirements, which represents a significant advancement for practical engineering applications. Traditional EVPS algorithms necessitate careful tuning of damping coefficients and weighting parameters for each specific problem type, whereas SA_EVPS automatically adapts these parameters during the optimization process based on continuous performance monitoring. The enhanced memory utilization in SA_EVPS addresses important limitations in traditional EVPS approaches by maintaining a diverse archive of good solutions rather than relying solely on the historically best solution. This approach actively preserves diversity and prevents premature convergence, with dynamic memory size adaptation ensuring appropriate memory utilization across problems of varying complexity levels.

The integration of statistical learning mechanisms enables SA_EVPS to effectively learn from successful solution patterns, a capability largely absent in standard EVPS implementations. This learning ability proves particularly valuable for large-scale problems where efficient navigation of high-dimensional search spaces is crucial for practical applicability. The statistical regeneration mechanism provides a principled approach to generating new solutions based on collective swarm intelligence, significantly enhancing the algorithm's performance on complex optimization landscapes. Additionally, improved constraint handling capabilities include advanced adaptive penalty methods and feasibility-preserving operators that maintain constraint satisfaction throughout the optimization process. These enhancements prove essential for practical structural optimization problems where constraint violations can lead to physically infeasible designs, with the constraint-guided search approach utilizing information from infeasible solutions to provide gradient information for constraint boundaries, thereby improving overall search efficiency.

Regarding computational complexity, SA_EVPS maintains linear complexity with respect to population size and problem dimensionality per iteration. The algorithm's efficiency stems from careful design of computational components and avoidance of expensive operations, with memory operations optimized through efficient data structures and statistical computations leveraging incremental update strategies to minimize computational overhead. This balance between computational cost and solution quality makes SA_EVPS particularly suitable for practical engineering applications where both factors represent critical considerations.

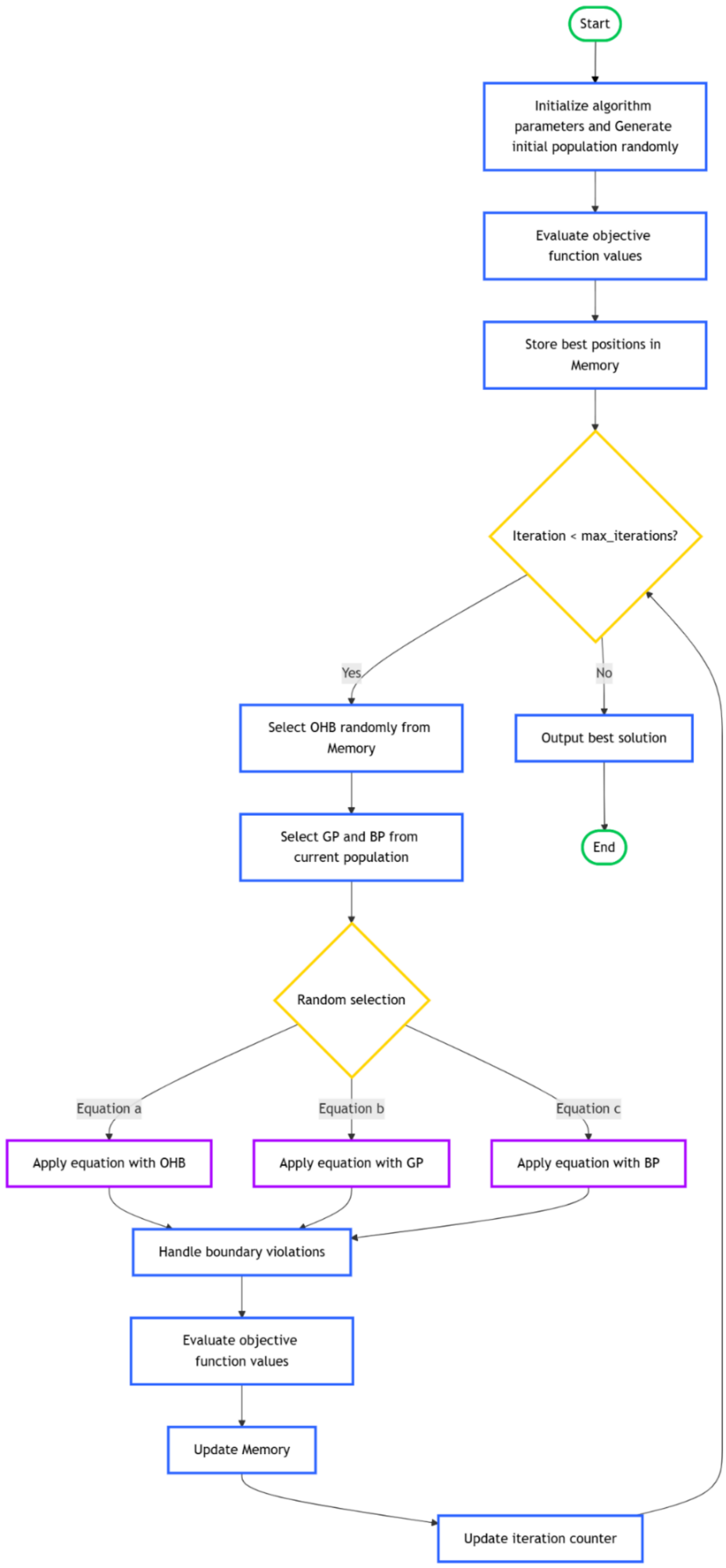


Figure 1: SA_EVPS Algorithm Flowchart

3. PROBLEM FORMULATION FOR DOME TRUSS OPTIMIZATION

3.1. Structural System Description

Dome truss structures represent three-dimensional spatial frameworks characterized by hemispherical or geodesic geometry with interconnected members forming efficient load-carrying systems. The optimization problem addresses simultaneous minimization of structural weight while satisfying natural frequency constraints that prevent resonance with external excitations and ensure acceptable dynamic behavior.

3.2. Mathematical Problem Formulation

The frequency-constrained dome optimization problem is formulated as a constrained nonlinear programming problem:

$$Minimize: f(X) = \sum_{i=1}^{m} \rho_i \times A_i \times L_i$$

Subject to:

$$\begin{aligned} & g_1(X):\ \omega_j(X) \geq \omega_{min,j} \ ;(j = 1, 2, ..., nf) \\ & g2(X):\ A_{min,i} \leq X_i \leq A_{max,i} \ ;(i = 1, 2, ..., n) \\ & h1(X):\ \left[K(X) - \omega_j{}^2 M(X)\right]\varphi_j = 0 \ ;(j = 1, 2, ..., nf) \end{aligned} \tag{1}$$

where:

- X **= [A_1, A_2, ..., A_n]**: Vector of cross-sectional areas (design variables).
- ρ_i, L_i : Material density and length of member i.
- ω_j : j-th natural frequency.
- $\omega_{min,j}$: Minimum required frequency for j-th mode.
- $K(X), M(X)$: Stiffness and mass matrices (functions of design variables).
- φ_j : j-th mode shape vector.
- **m, n, nf**: Number of members, design variables, and frequency constraints.

3.3. Benchmark Problem Configurations

Three benchmark dome truss structures with varying complexity levels are investigated to evaluate the SA_EVPS algorithm performance under frequency constraints. The optimization process involves 20 independent runs for each structure to ensure statistical validity. The structural configurations include: (1) a 600-bar single-layer dome with 25 design variable groups, (2) an 1180-bar single-layer dome with 59 design variable groups, and (3) a 1410-bar double-layer dome with 47 design variable groups.

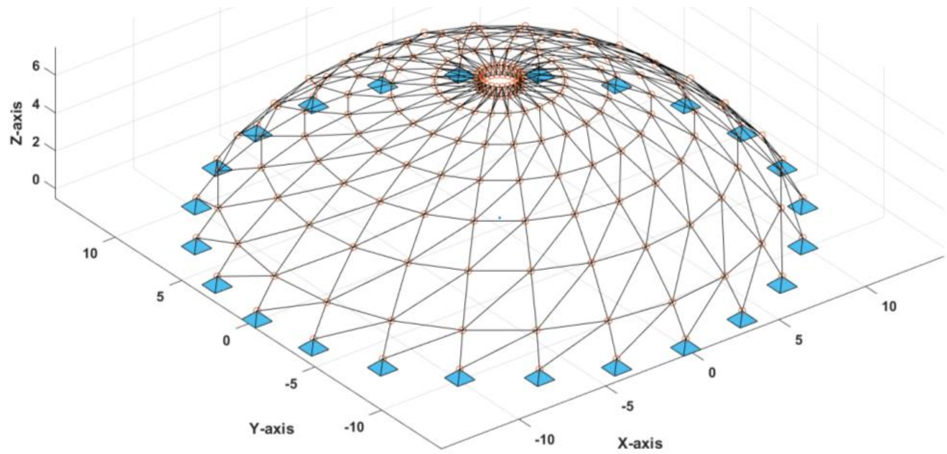
For all structures, the following properties are adopted:

- Elastic modulus: $E = 2.0 \times 10^{11}$ N/m²
- Material density: $\rho = 7850$ kg/m³

- Non-structural mass: 100 kg attached to each free node
- Cross-sectional area bounds: 1.0×10^{-4} m² ≤ A_i ≤ 1.0×10^{-2} m²
- Objective function: Minimization of total structural weight with frequency constraints

3.3.1. Benchmark Problem Configurations

The distinct structural configuration obtained through optimization is comprehensively illustrated in Figure 2. The three-dimensional layout (Figure 2(a)), X-Y plane projection (Figure 2(b)), and X-Z plane projection (Figure 2(c)) provide complete geometric understanding, while Figure 2(d) presents the three-dimensional view with detailed node numbering for comprehensive structural identification.



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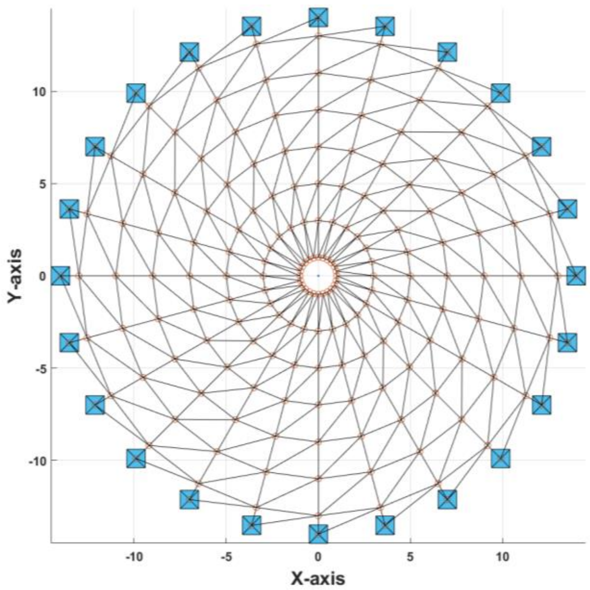

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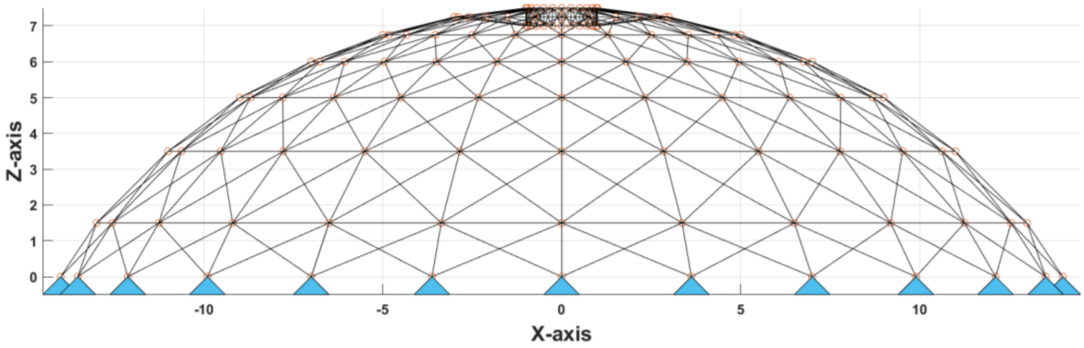

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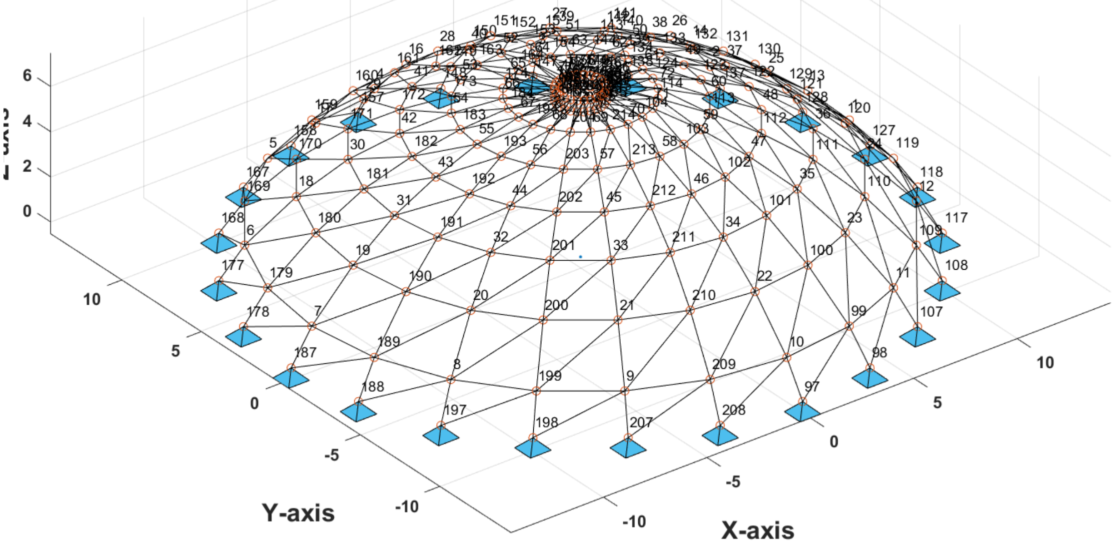


(d)

Figure 2: Geometric configuration of the 600-bar single-layer dome structure: (a) Three-dimensional view of the optimized structure, (b) X-Y plane projection with node numbers, (c) X-Z plane projection with node numbers, (d) Three-dimensional view with complete node numbering

The 600-bar dome structure represents a medium-scale benchmark problem that has been extensively studied in structural optimization literature [19-21]. This single-layer dome configuration presents significant computational challenges due to its geometric complexity and frequency constraints, making it an ideal test case for evaluating the performance of the proposed SA_EVPS algorithm. The dome structure comprises 216 nodes and 600 structural members organized in a sophisticated geometric pattern. The Cartesian coordinates of all nodes are systematically presented in Table 1, providing complete geometric specification for reproducibility. The structural layout follows a modular construction approach with 30 standard modules separated by 15° angular increments, ensuring symmetric load distribution and mechanical efficiency.

Table 1: Node Coordinates for the 600-Bar Single-Layer Dome Structure

Node number	Coordinates (x, y, z) (m)
1	(1.0, 0.0, 7.0)
2	(1.0, 0.0, 7.5)
3	(3.0, 0.0, 7.25)
4	(5.0, 0.0, 6.75)
5	(7.0, 0.0, 6.0)
6	(9.0, 0.0, 5.0)
7	(11.0, 0.0, 3.5)
8	(13.0, 0.0, 1.5)
9	(14.0, 0.0, 0.0)

The frequency constraints imposed on the optimization process require the first natural frequency (ω_1) to be ≥ 5 Hz and the third natural frequency (ω_3) to be ≥ 7 Hz. These constraints ensure adequate dynamic performance and vibration resistance under operational conditions.

The SA_EVPS algorithm demonstrated exceptional performance in optimizing the 600-bar dome structure. Table 2 presents comprehensive comparative results against five established optimization algorithms: DPSO [22], CBO [23], ECBO [23], VPS [24], and standard EVPS [25]. The SA_EVPS algorithm achieved the best optimized weight of 6065.72 kg, representing a significant improvement over previous approaches.

Table 2: Optimized Cross-Sectional Areas (cm²) and Weight Comparison for the 600-Bar Dome

Element No. (nodes)	DPSO [22]	CBO [23]	ECBO [23]	VPS [24]	EVPS [25]	This study
1 (1-2)	1.365	1.2404	1.4305	1.3155	1.2019	1.3520
2 (1-3)	1.391	1.3797	1.3941	1.2299	1.5012	1.3885
3 (1-10)	5.686	5.2597	5.5293	5.5506	5.3603	5.3824
4 (1-11)	1.511	1.2658	1.0469	1.3867	1.2323	1.3866
5 (2-3)	17.711	17.2255	16.9642	17.4275	16.8524	17.8464
6 (2-11)	36.266	38.2991	35.1892	40.143	35.879	36.1427
7 (3-4)	13.263	12.2234	12.2171	12.8848	12.9692	12.8401
8 (3-11)	16.919	15.4712	16.7152	15.5413	15.799	15.2408
9 (3-12)	13.333	11.1577	12.5999	12.2428	10.7142	10.8233
10 (4-5)	9.534	9.4636	9.5118	9.3776	9.0974	8.9136
11 (4-12)	9.884	8.825	8.9977	8.6684	8.081	8.4160
12 (4-13)	9.547	9.1021	9.4397	9.1659	9.228	8.8574
13 (5-6)	7.866	6.8417	6.8864	7.1664	7.4727	7.2142
14 (5-13)	5.529	5.2882	4.2057	5.217	5.4983	5.1157
15 (5-14)	7.007	6.7702	7.2651	6.5346	6.466	6.3966
16 (6-7)	5.462	5.1402	6.1693	5.4741	5.0321	5.2589

17 (6-14)	3.853	5.1827	3.9768	3.6545	3.5817	3.5547
18 (6-15)	7.432	7.4781	8.3127	7.6034	7.7686	7.8748
19 (7-8)	4.261	4.5646	4.1451	4.2251	4.619	4.2863
20 (7-15)	2.253	1.8617	2.4042	1.9717	2.2625	2.2738
21 (7-16)	4.337	4.8797	4.3038	4.5107	4.4862	4.7653
22 (8-9)	4.028	3.5065	3.2539	3.5251	3.4169	3.5519
23 (8-16)	1.954	2.4546	1.8273	1.9255	1.7917	1.8090
24 (8-17)	4.709	4.9128	4.8805	4.7628	4.7613	4.8001
25 (9-17)	1.41	1.2324	1.5276	1.6854	1.6376	1.6251
Best weight (kg)	6344.55	6182.01	6171.51	6120.01	6067.74	6065.72
Average optimized weight (kg)	6674.71	6226.37	6191.5	6158.11	6069.69	6078.14
Standard deviation (kg)	473.21	60.12	39.08	28.49	3.02	11.26

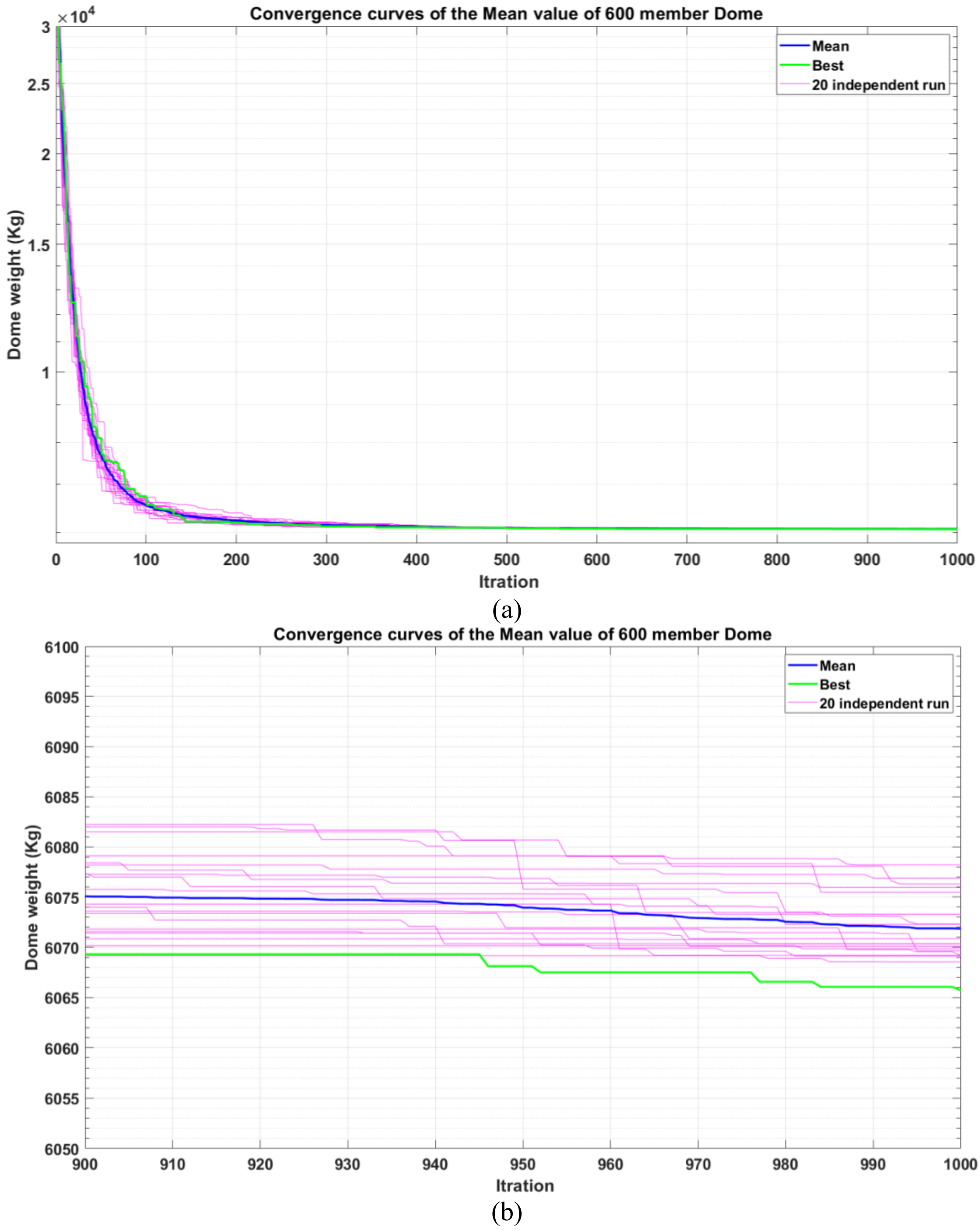
The frequency constraint satisfaction results, as shown in Table 3, confirm that SA_EVPS successfully met all design requirements with ω_1 = 5.00088 Hz and ω_3 = 7.00058 Hz, demonstrating precise constraint handling capability.

Table 3: Natural Frequency Constraints Verification for the 600-Bar Dome

Structures	Frequencies	Minimum Permitted value	This study
600-bar	ω_1	5Hz	5.00088
dome	ω_3	7Hz	7.00058

The optimization results for the 600-bar single-layer dome demonstrate the exceptional performance of the SA-EVPS algorithm. Achieving the best optimized weight of **6,065.72 kg**, SA-EVPS outperformed all comparative methods, showing marginal but consistent improvement over EVPS (6,067.74 kg) and significant advantages over VPS (6,120.01 kg), ECBO (6,171.51 kg), CBO (6,182.01 kg), and DPSO (6,344.55 kg). The statistical analysis across 20 independent runs reveals outstanding consistency with an average weight of 6,078.14 kg. While standard EVPS achieved the lowest standard deviation (3.02 kg), SA-EVPS's combination of superior minimum weight and excellent stability validates the effectiveness of its self-adaptive enhancements.

The convergence characteristics, illustrated in Figure 3, demonstrate the algorithm's efficient search capability. Figure 3(a) shows consistent convergence patterns across 20 independent runs, while Figure 3(b) provides a detailed view of iterations 900-1000, confirming the algorithm's stability and precision in the final optimization phase.



(a)

(b)

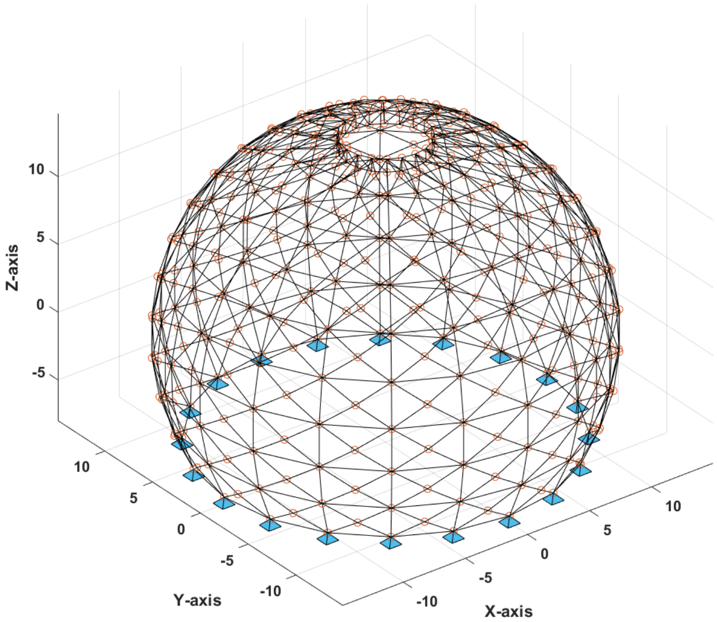
Figure 3: Convergence characteristics of the SA_EVPS algorithm for the 600-bar dome optimization: (a) Convergence history for 20 independent runs over 1000 iterations, (b) Detailed view of the final convergence phase (iterations 900-1000) showing algorithmic stability

The optimization process employed a population size of 50 particles with a maximum of 1000 structural analyses per run, ensuring computational efficiency while maintaining solution quality. The repeated experiments across 20 independent runs demonstrated the algorithm's consistency and reliability in handling complex structural optimization problems with frequency constraints.

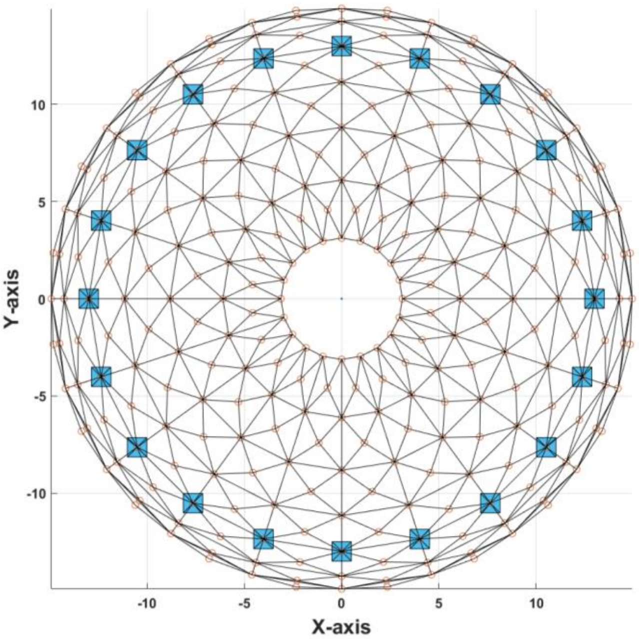
3.3.2. 1180-Bar Single-Layer Dome

The 1180-bar dome structure represents a large-scale single-layer configuration that presents significant computational challenges in structural optimization. This benchmark problem has been extensively studied in the literature [34-38] and serves as a rigorous test case for evaluating the performance of advanced optimization algorithms. The structure's complexity and scale make it an ideal candidate for demonstrating the capabilities of the proposed SA_EVPS algorithm in handling large-scale structural optimization problems with frequency constraints.

The 1180-bar single-layer dome features a sophisticated geometric arrangement with 400 nodes and 1180 structural members. The structure follows a modular design approach with 20 substructures, each containing 20 nodes and 59 elements. The angular separation between adjacent substructures is 18°, ensuring symmetric load distribution and structural integrity. Figure 4 illustrates the complete geometric configuration through multiple perspectives: (a) three-dimensional view of the optimized structure, (b) X-Y plane projection, (c) X-Z plane projection, and (d) three-dimensional view with comprehensive node numbering for structural element identification.



(a)



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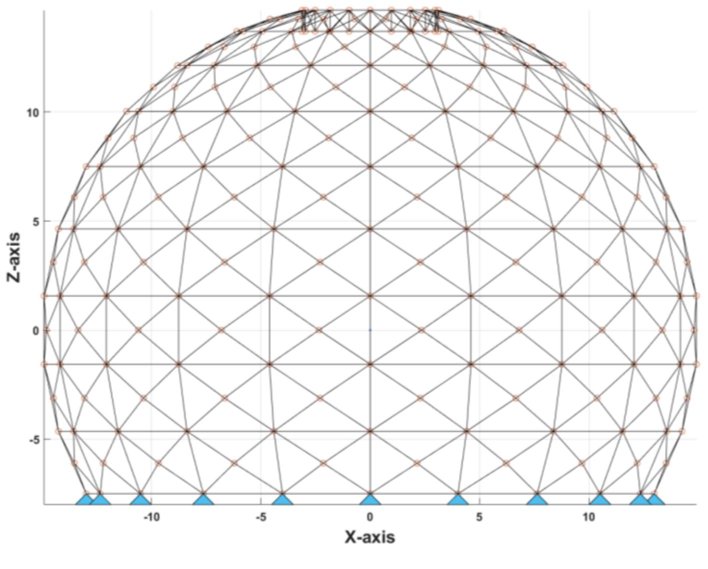

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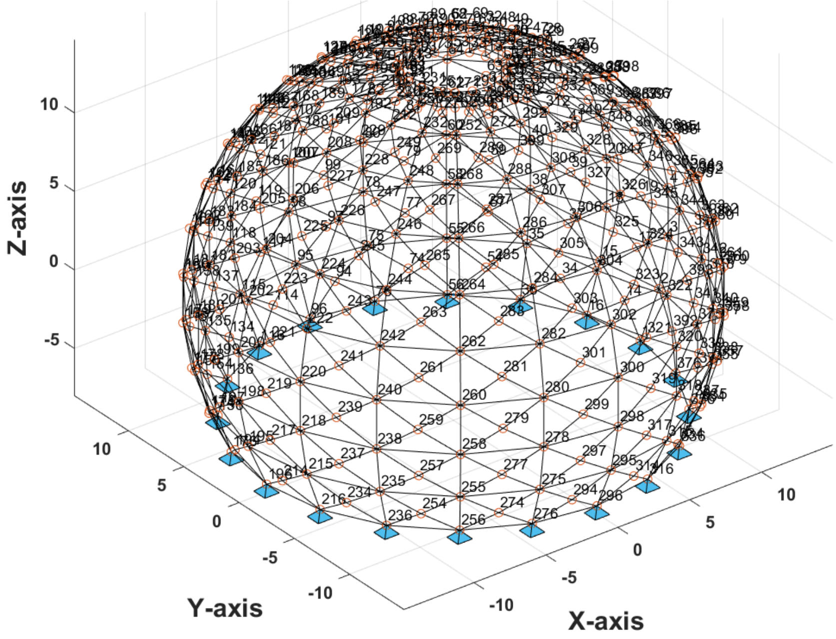


(d)

Figure 4: Geometric configuration of the 1180-bar single-layer dome structure: (a) Three-dimensional view of the optimized structure, (b) X-Y plane projection with node numbers, (c) X-Z plane projection with node numbers, (d) Three-dimensional view with complete node numbering

The Cartesian coordinates of the primary nodes are systematically presented in Table 4, providing the essential geometric specification for structural analysis and optimization. The dome's geometry follows a spherical configuration with varying radial distances and vertical positions, creating an efficient load-bearing structure suitable for large-span applications.

Table 4: Coordinates of the Primary Nodes of the 1180-Bar Dome

Node number	Coordinates (x, y, z) (m)	Node number	Coordinates (x, y, z) (m)
1	(3.1181, 0.0, 14.6723)	11	(4.5788, 0.7252, 14.2657)
2	(6.1013, 0.0, 13.7031)	12	(7.4077, 1.1733, 12.9904)
3	(8.8166, 0.0, 12.1354)	13	(9.9130, 1.5701, 11.1476)
4	(11.1476, 0.0, 10.0365)	14	(11.9860, 1.8984, 8.8165)
5	(12.9904, 0.0, 7.5000)	15	(13.5344, 2.1436, 6.1013)
6	(14.2657, 0.0, 4.6358)	16	(14.4917, 2.2953, 3.1180)
7	(14.9179, 0.0, 1.5676)	17	(14.8153, 2.3465, 0.0)
8	(14.9179, 0.0, -1.5677)	18	(14.9179, 2.2953, -3.1181)
9	(14.2656, 0.0, -4.6359)	19	(13.5343, 2.1436, -6.1014)
10	(12.9903, 0.0, -7.5001)	20	(3.1181, 0.0, 13.7031)

The frequency constraints imposed on the structure require:

- First natural frequency: **$\omega_1 \geq 7$ Hz**
- Third natural frequency: **$\omega_3 \geq 9$ Hz**

These constraints ensure adequate dynamic performance and vibration resistance under operational conditions, particularly important for large-scale dome structures subject to environmental loads.

The SA_EVPS algorithm demonstrated superior performance in optimizing the 1180-bar dome structure. Table 5 presents comprehensive comparative results against five established optimization algorithms: DPSO [34], CBO [35], ECBO [37], VPS [38], and standard EVPS. The optimized cross-sectional areas for all 59 design variables are provided, along with statistical performance metrics.

Table 5: Optimized Cross-Sectional Areas (cm²) and Weight Comparison for the 1180-Bar Dome

Element No. (nodes)	DPSO [22]	CBO [23]	ECBO [23]	VPS [24]	EVPS [25]	This study
1 (1-2)	7.926	13.0171	7.6678	6.8743	8.2704	7.757332
2 (1-11)	10.426	10.4346	11.1437	10.023	9.0477	9.442601
3 (1-20)	2.115	3.0726	1.852	4.414	2.4083	2.275153
4 (1-21)	14.287	12.6969	14.5563	13.5515	17.6548	15.0952
5 (1-40)	3.846	3.5654	4.9499	1.8303	5.0107	3.953689
6 (2-3)	5.921	6.519	6.8095	7.0824	6.8212	5.983274
7 (2-11)	7.955	7.4233	6.6803	6.396	5.5067	6.407329
8 (2-12)	6.697	6.3471	6.7889	6.5646	6.3639	6.466585
9 (2-20)	1.889	2.3013	1.063	2.3705	2.3437	1.892457
10 (2-22)	11.881	12.1936	9.1602	13.2621	11.4931	10.79132
11 (3-4)	7.121	7.2877	6.9891	7.0922	7.6098	7.074334
12 (3-12)	6.08	7.0961	6.9881	6.8079	6.0982	5.352509
13 (3-13)	6.599	6.5669	6.9555	6.3815	6.5201	7.607012
14 (3-23)	7.772	7.8257	7.5443	7.3122	7.6463	7.406591
15 (4-5)	9.358	8.6812	9.5431	8.7221	10.5859	8.729909
16 (4-13)	6.213	5.7888	6.9123	6.368	6.378	6.845167
17 (4-14)	8.2	21.1342	8.9891	7.3159	7.0271	9.473977
18 (4-24)	7.799	10.0502	6.8926	11.5749	7.9658	7.781129
19 (5-6)	11.752	12.9279	12.6128	14.7985	12.0607	11.37844
20 (5-14)	7.494	9.3212	8.1983	5.5174	8.398	8.191488
21 (5-15)	9.696	10.126	11.8358	15.7381	12.5623	10.31285
22 (5-25)	9.177	10.1358	9.7321	8.3419	9.2832	8.780953
23 (6-7)	17.326	15.8585	19.165	17.5	17.0449	18.05872
24 (6-15)	11.797	9.9672	10.4682	10.3084	11.0122	9.917823
25 (6-16)	14.002	14.8493	14.1178	15.1958	14.768	14.09229
26 (6-26)	11.562	11.4909	11.14567	10.9395	10.9959	11.8396
27 (7-8)	23.981	26.2359	23.4125	24.9421	21.915	23.36898
28 (7-16)	12.996	13.8812	15.5167	13.9614	16.1827	12.87851
29 (7-17)	16.591	18.8857	16.6613	18.4153	20.8216	16.7722
30 (7-27)	15.91	14.0257	15.9631	14.4945	18.0993	15.7957
31 (8-9)	34.642	33.8826	37.0532	36.3529	34.2502	33.48006
32 (8-17)	19.86	25.7142	22.2937	19.6608	16.8668	19.78771
33 (8-18)	25.079	24.8644	22.7409	23.7259	25.6881	22.11768
34 (8-28)	18.965	19.8498	23.5624	22.0297	22.372	22.34356
35 (9-10)	47.514	53.263	47.7652	47.3286	46.6206	48.42987
36 (9-18)	28.133	22.7771	22.5066	22.9442	22.0552	25.10096
37 (9-19)	33.023	35.423	34.6418	30.8229	28.437	33.04163
38 (9-29)	32.263	57.548	31.6492	33.1098	29.5337	32.88533
39 (10-19)	33.401	35.1385	32.7268	32.5526	36.8086	37.07061

40 (10-30)	1.344	10.73	1.05206	1.7363	1.5819	1.093214
41 (11-21)	9.327	9.2401	11.3681	11.5271	8.7834	9.371286
42 (11-22)	7.202	5.2661	6.5512	8.4571	6.3483	6.719762
43 (12-22)	6.792	6.2415	6.3619	5.4136	5.6296	6.732481
44 (12-23)	6.228	4.4768	5.9296	7.1832	5.6345	5.61487
45 (13-23)	6.601	8.8846	7.8739	5.4066	6.4783	6.113859
46 (13-24)	6.584	7.371	6.2794	6.2534	6.3008	6.354765
47 (14-24)	8.32	8.2595	7.6206	6.9383	8.7835	9.692346
48 (14-25)	8.844	7.6091	7.2937	10.6872	7.3685	7.68342
49 (15-25)	11.254	11.303	10.5783	12.8005	11.9231	10.6635
50 (15-26)	12.162	13.8381	10.1173	10.2216	10.1779	11.74101
51 (16-26)	13.854	13.3654	15.1088	11.533	13.9712	13.92774
52 (16-27)	13.844	13.1836	12.8251	11.6918	13.125	14.49242
53 (17-27)	17.536	13.5793	17.4375	20.7566	17.0503	17.30932
54 (17–28)	20.551	10.0628	20.1153	18.1341	23.0291	19.08312
55 (18–28)	24.072	24.1197	24.2121	28.2882	23.4632	22.90531
56 (18–29)	27.287	24.2604	23.3175	24.2023	23.4265	24.84708
57 (19–29)	32.965	34.1389	34.6196	48.018	35.9923	33.66983
58 (19–30)	36.94	38.034	35.297	35.6517	42.7071	34.7992
59 (20–40)	3.837	2.6689	8.8569	5.5956	5.0568	5.600315
Best weight (kg)	37779.81	40985	37984.39	38699.14	38008.9	37492.63
Average optimized weight (kg)	38294.45	42019.1	38042.15	38861.82	38171.95	37522.61
Standard deviation (kg)	550.5	655.72	101.43	385.41	139.41	48.36733

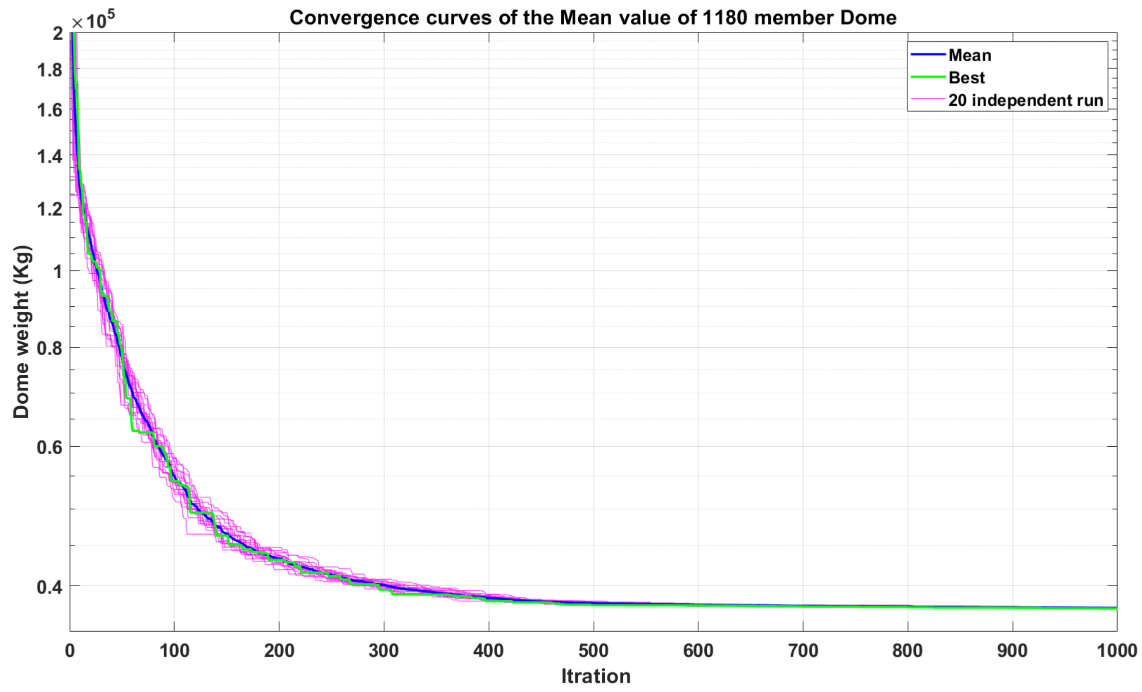
For the more complex 1180-bar single-layer dome with 59 design variable groups, SA-EVPS achieved the best optimized weight of **37,492.63 kg**, representing substantial improvements across all comparative algorithms: 0.8% better than DPSO (37,779.81 kg), 1.4% better than EVPS (38,008.9 kg), 3.1% better than VPS (38,699.14 kg), and 8.5% better than CBO (40,985 kg). The algorithm's robustness is particularly evident in its statistical performance, with an average weight of 37,522.61 kg and an exceptionally low standard deviation of 48.37 kg across 20 runs. This stability represents dramatic improvements: 91.2% reduction compared to DPSO (σ = 550.5 kg), 92.6% reduction compared to CBO (σ = 655.72 kg), and 65.3% reduction compared to standard EVPS (σ = 139.41 kg). These results confirm SA-EVPS's ability to consistently find high-quality solutions even as problem complexity increases significantly.

The frequency constraint satisfaction results, presented in Table 6, confirm that SA_EVPS successfully met all design requirements with high precision:

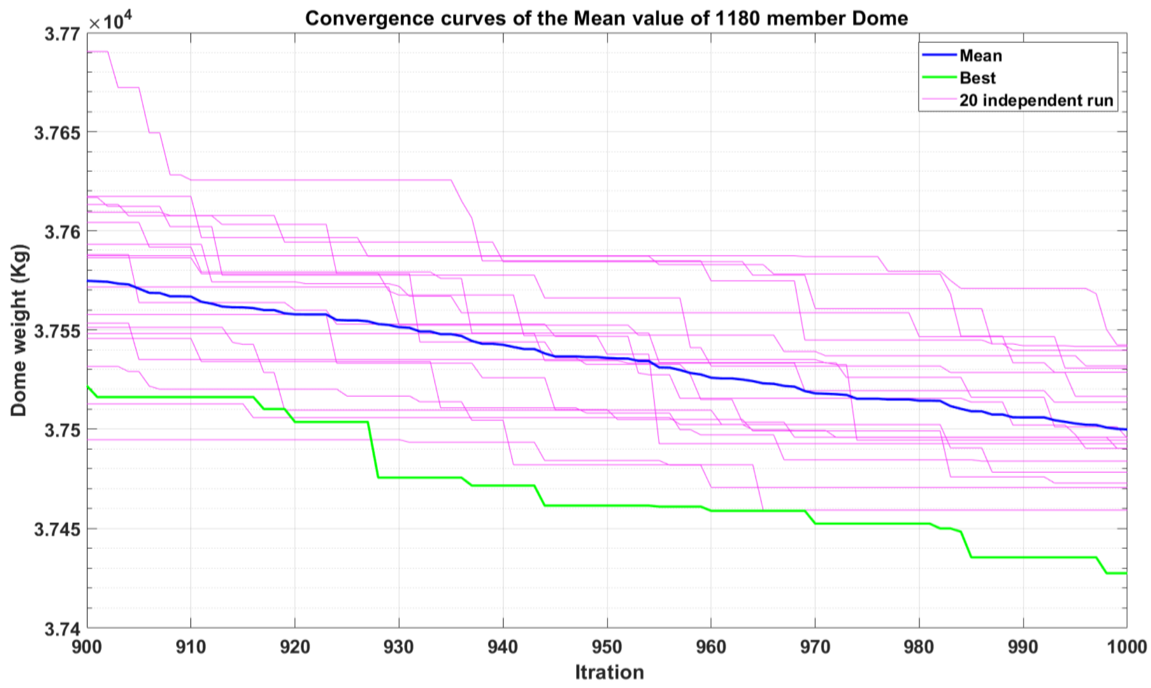
Table 6: Natural Frequency Constraints Verification for the 1180-Bar Dome

Structures	Frequencies	Minimum Permitted value	This study
1180-bar dome	$\omega 1$	7Hz	7.00066
	$\omega 3$	9Hz	9.00297

The convergence characteristics, illustrated in Figure 5, demonstrate the algorithm's efficient search capability and stability. Figure 5(a) shows the convergence history for 20 independent runs over 1000 iterations, revealing consistent convergence patterns with minimal variation. Figure 5(b) provides a detailed view of iterations 900-1000, confirming the algorithm's precision and stability in the final optimization phase. The enhanced version of the VPS showed better performance in terms of best and average weight, and it converged rapidly according to the convergence curve displayed in Figure 4.



(a)



(b)

Figure 5: Convergence characteristics of the SA_EVPS algorithm for the 1180-bar dome optimization: (a) Convergence history for 20 independent runs over 1000 iterations, (b) Detailed view of the final convergence phase (iterations 900-1000) showing algorithmic stability

The optimization process employed a population size of 70 particles with a maximum of 1000 structural analyses per run. This configuration balanced computational efficiency with solution quality, enabling thorough exploration of the design space while maintaining reasonable computational costs. The experiment was repeated 20 times to ensure statistical validity and demonstrate the algorithm's consistency.

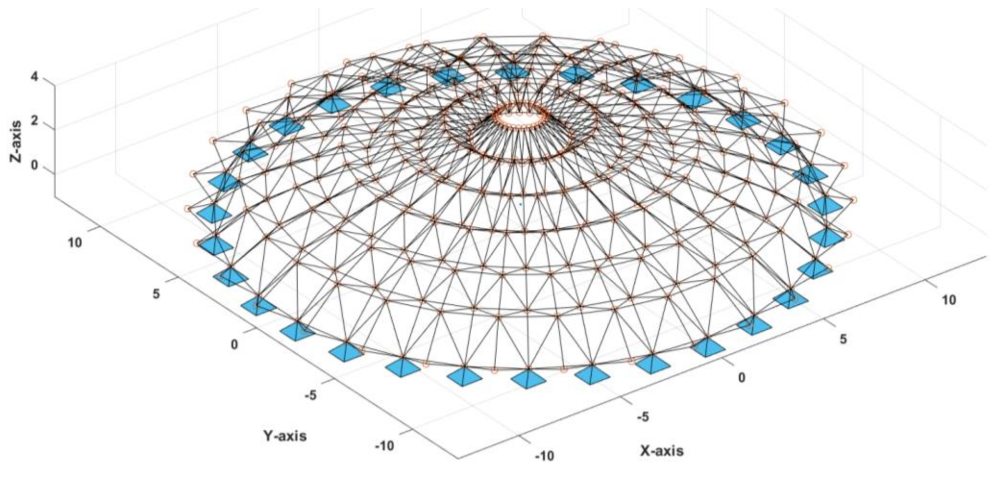
The successful optimization of the 1180-bar dome structure validates the SA_EVPS algorithm's capability in handling large-scale structural optimization problems with multiple frequency constraints. The combination of superior optimization performance, exceptional consistency, and precise constraint satisfaction establishes SA_EVPS as a highly effective approach for complex structural optimization challenges in engineering practice.

3.3.3. 1410-Bar Double-Layer Dome

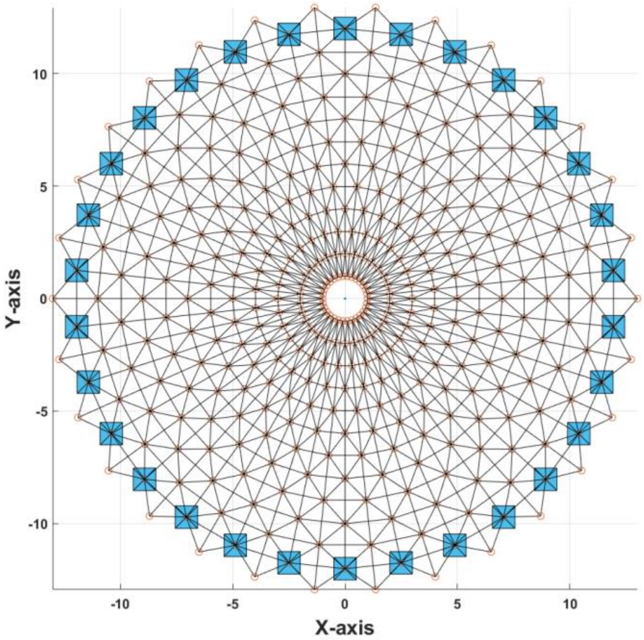
The 1410-bar double-layer dome represents the most complex benchmark structure in this study, featuring an intricate double-layer configuration that provides superior structural performance and redundancy. This advanced structural system has been extensively investigated in the optimization literature [34-38] and serves as a comprehensive test case for evaluating the capabilities of optimization algorithms in handling highly complex structural systems with stringent frequency constraints.

The 1410-bar dome features a sophisticated double-layer geometry comprising 390 nodes and 1410 structural members. The double-layer configuration provides enhanced load distribution capabilities and structural redundancy, making it particularly suitable for large-span applications requiring high reliability. The structure consists of 13 nodes and 47 elements per substructure, with an angular separation of 12° between adjacent substructures, creating a symmetric and efficient load-bearing system.

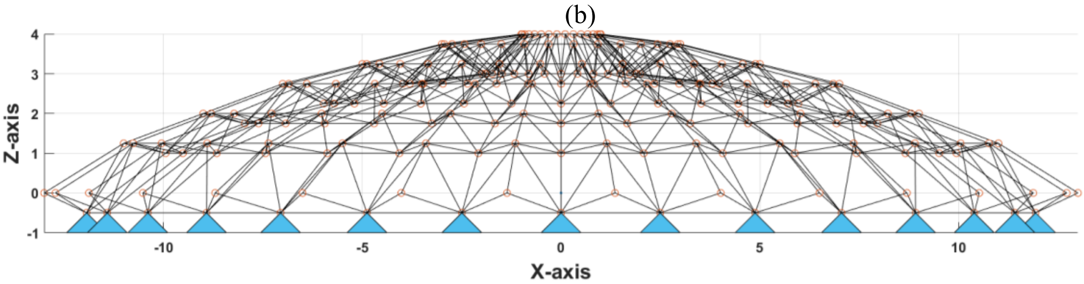
Figure 6 presents the complete geometric configuration through three distinct views: (a) three-dimensional perspective of the double-layer structure, (b) plan view showing the radial arrangement and node distribution, and (c) elevation view illustrating the dome's vertical profile and layer separation. The double-layer system creates a complex network of interconnected members that work together to provide exceptional structural performance.



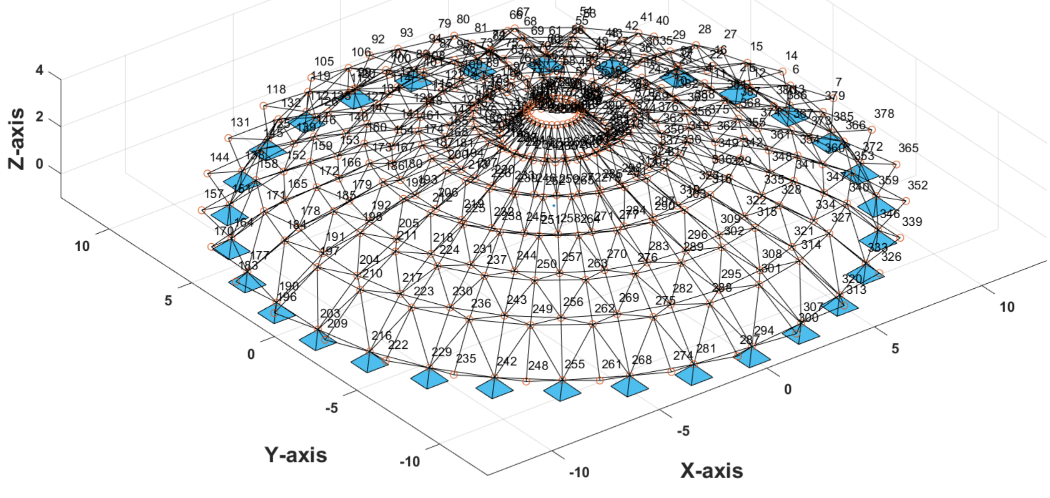
(a)



(b)



(c)



(d)

Figure 6: Geometric configuration of the 1410-bar double -layer dome structure: (a) Three-

dimensional view of the optimized structure, (b) X-Y plane projection with node numbers, (c) X-Z plane projection with node numbers, (d) Three-dimensional view with complete node numbering

The Cartesian coordinates of the primary nodes defining the structural geometry are systematically presented in Table 7, providing essential geometric specifications for structural analysis and optimization.

Table 7. Node Coordinates for the 1410-Bar Double-Layer Dome Structure

Node No.	(x,y,z)	Node No.	(x,y,z)
1	(1.0, 0.0, 4.0)	8	(1.989, 0.209, 3.0)
2	(3.0, 0.0, 3.75)	9	(3.978, 0.418, 2.75)
3	(5.0, 0.0, 3.25)	10	(5.967, 0.627, 2.25)
4	(7.0, 0.0, 2.75)	11	(7.956, 0.836, 1.75)
5	(9.0, 0.0, 2.0)	12	(9.945, 1.0453, 1.0)
6	(11.0, 0.0, 1.25)	13	(11.934, 1.2543, - 0.5)
7	(13.0, 0.0, 0.0)		

The double-layer configuration creates two interconnected shell surfaces with diagonal and vertical members providing the connection between layers. This arrangement significantly enhances the structure's resistance to buckling and improves its overall stability under various loading conditions.

The frequency constraints imposed on the structure require:

- First natural frequency: $\omega_1 \geq 7$ **Hz**
- Third natural frequency: $\omega_3 \geq 9$ **Hz**

These constraints are particularly challenging for double-layer structures due to the complex dynamic behavior arising from the interaction between the two layers and the connecting members.

The SA_EVPS algorithm demonstrated exceptional performance in optimizing the 1410-bar double-layer dome. Table 8 presents comprehensive comparative results against five established optimization algorithms, including the optimized cross-sectional areas for all 47 design variables and statistical performance metrics.

Table 8: Optimized Cross-Sectional Areas (cm²) and Weight Comparison for the 1410-Bar Dome

Element No. (nodes)	DPSO [22]	CBO [23]	ECBO [23]	VPS [24]	EVPS [25]	This study
1 (1-2)	7.209	1.0073	7.7765	5.6333	6.9338	6.0790
2 (1-8)	5.006	2.5808	6.2173	4.7628	4.7701	4.8829
3 (1-14)	38.446	24.3407	23.9162	37.7385	29.4676	29.2239
4 (2-3)	9.438	6.675	11.2399	7.4927	10.3698	8.8991
5 (2-8)	4.313	3.8881	2.5775	3.1824	5.8838	6.9283
6 (2-9)	1.494	5.0607	1.8559	1.0193	2.0475	2.0068
7 (2-15)	8.455	78.9781	16.9202	8.9475	15.0685	18.0076

8 (3-4)	9.488	9.2944	13.7947	10.4272	9.187	10.0911
9 (3-9)	3.48	2.6585	5.4502	4.1398	2.5231	2.2439
10 (3-10)	3.495	3.5399	2.9751	3.1408	3.1458	2.9075
11 (3-16)	16.037	10.2473	13.7811	15.4194	8.5578	10.8098
12 (4-5)	9.796	9.682	9.387	8.9931	9.0714	9.2720
13 (4-10)	2.413	2.4435	2.3499	3.1988	2.0449	2.5421
14 (4-11)	5.681	5.0637	4.9125	7.1565	4.452	4.3725
15 (4-17)	15.806	12.9434	11.8755	17.8564	15.5304	13.4310
16 (5-6)	8.078	6.9073	8.8668	9.2685	8.0463	8.1039
17 (5-11)	3.931	3.1808	3.6304	3.3221	4.1273	2.7187
18 (5-12)	6.099	5.9622	6.2651	6.1486	5.8742	6.2456
19 (5-18)	10.771	13.3195	15.103	8.4422	12.2753	13.8780
20 (6-7)	13.775	13.2136	13.1091	12.8578	13.8096	13.7746
21 (6-12)	4.231	5.4405	5.294	5.8031	5.5497	5.7483
22 (6-13)	6.995	8.4703	5.9929	7.5484	7.8487	7.1347
23 (6-19)	1.837	1.87	1	1.4805	1.2083	1.0723
24 (7-13)	4.397	5.5203	4.9879	4.5332	4.4281	4.1662
25 (8-9)	2.115	2.4492	3.178	2.0347	3.4544	2.8697
26 (8-14)	4.923	2.215	5.9226	5.8589	4.7012	4.5996
27 (8-15)	4.047	3.1193	2.4607	2.4401	6.5027	6.3020
28 (8-21)	5.906	8.7508	7.571	6.925	14.0563	12.4698
29 (9-10)	3.392	5.1195	4.8616	3.3875	3.754	3.7487
30 (9-15)	1.902	3.8508	1.5956	1.5024	1.8509	1.5852
31 (9-16)	4.381	4.4435	4.9084	4.0498	3.643	2.2556
32 (9-22)	8.442	9.1339	11.6118	11.0886	4.6275	3.9184
33 (10-11)	5.011	5.7811	5.2554	5.4639	6.1824	5.4267
34 (10-16)	3.577	3.451	2.8687	2.8459	2.6757	3.4337
35 (10-17)	2.805	1.8344	2.3286	2.3136	2.2184	2.6045
36 (10-23)	2.024	2.7952	1.6159	3.437	1.2067	1.5812
37 (11-12)	6.709	7.2668	6.9795	8.0225	6.8483	6.6815
38 (11-17)	5.054	4.7761	5.3159	5.8009	4.0047	4.1946
39 (11-18)	3.259	3.3394	2.9915	4.4004	3.8577	3.7762
40 (11-24)	1.063	1.0001	1.0018	1.0005	1.2502	1.1001
41 (12-13)	5.934	7.3874	4.1091	7.7222	5.7831	6.3622
42 (12-18)	7.057	7.3114	6.013	5.2574	5.6696	5.9529
43 (12-19)	5.745	4.8773	5.8695	4.5055	6.2424	4.9815
44 (12-25)	1.185	1	1	1.0005	1.6216	1.0485
45 (13-19)	7.274	7.9928	7.7041	7.9383	6.6286	8.2356
46 (13-20)	4.798	3.4989	3.76	4.7805	4.6648	4.1771
47 (13-26)	1.515	2.0951	1.0006	1.0054	1.0336	1.0614
Best weight (kg)	10453.84	11102.84	10739.19	10491.83	10391.51	10310.25
Average optimized weight (kg)	11100.57	12359.41	10812.2	10936.34	10412.82	10383.65
Standard deviation (kg)	334.2	251.88	64.91	158.39	41.1	55.95

The 1410-bar double-layer dome represents the most structurally complex test case, where SA-EVPS achieved the best optimized weight of **10,310.25 kg**. This result demonstrates improvements of 1.4% over DPSO (10,453.84 kg), 7.1% over CBO (11,102.84 kg), 4.0% over ECBO (10,739.19 kg), 1.7% over VPS (10,491.83 kg), and 0.8% over standard EVPS (10,391.51 kg). The statistical analysis shows strong consistency with an average weight of 10,383.65 kg and a standard deviation of 55.95 kg. While this standard deviation is slightly higher than EVPS (41.1 kg), representing a 36.2% increase, it remains significantly better than DPSO (83.3% reduction), CBO (77.8% reduction), and VPS (64.7% reduction). The consistent achievement of low weights across all runs, despite the structural complexity of the double-layer configuration, validates the algorithm's effectiveness in handling sophisticated dome optimization problems with multiple design variables and frequency constraints.

While the standard EVPS showed slightly better consistency, the SA_EVPS achieved superior optimization performance with acceptable variability, striking an optimal balance between exploration and exploitation.

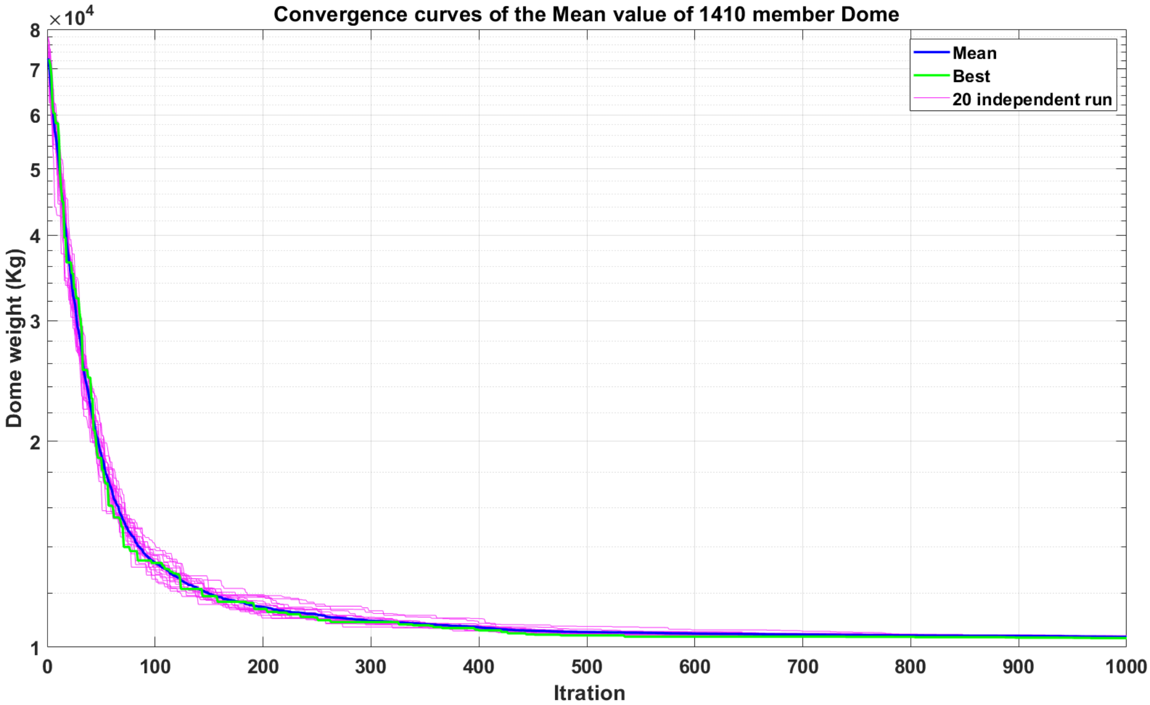
The frequency constraint verification results, presented in Table 9, confirm that SA_EVPS successfully satisfied all design requirements with high precision:

Table 9: Natural Frequency Constraints Verification for the 1410-Bar Dome

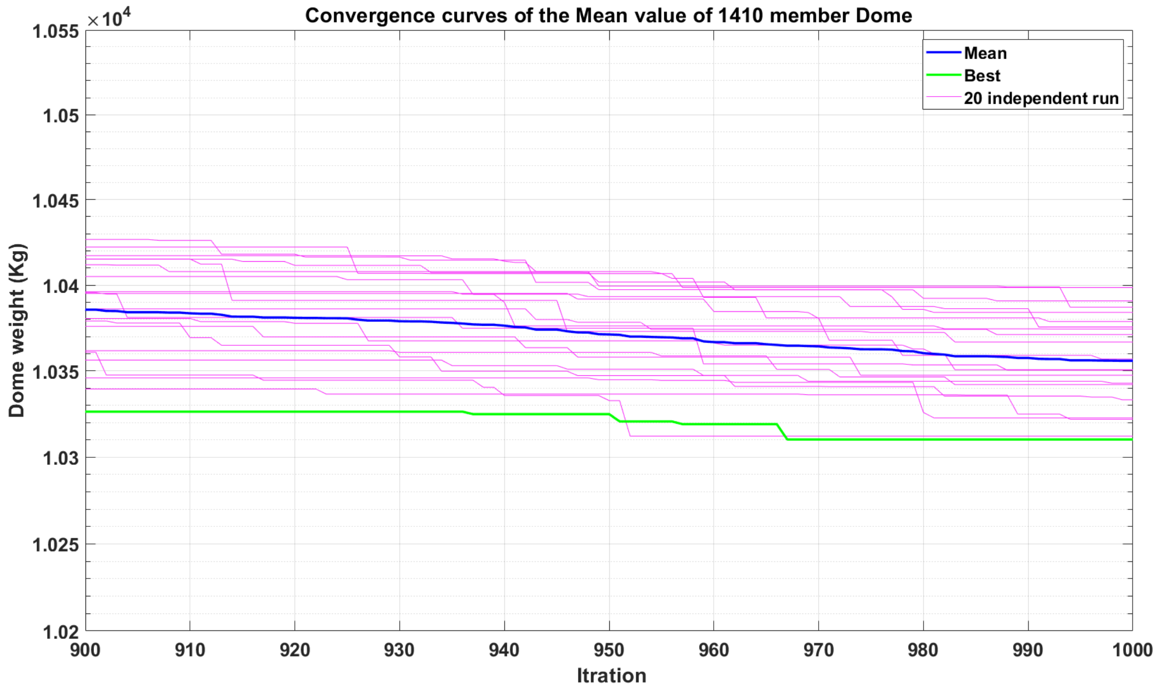
Structures	Frequencies	Minimum Permitted value	This study
1410-bar dome	ω1	7Hz	7.00398
	ω3	9Hz	9.00106

The precise satisfaction of frequency constraints (within 0.06% and 0.01% of the limits, respectively) demonstrates the algorithm's exceptional constraint-handling capability, crucial for practical engineering applications where frequency requirements are critical for structural performance.

Figure 7 illustrates the convergence behavior of the SA_EVPS algorithm for the 1410-bar dome optimization. The convergence curves reveal rapid initial progress followed by steady refinement, with the enhanced version of the VPS showing better performance in terms of best and average weight. The algorithm converged rapidly according to the convergence curve displayed in Figure 6, typically reaching near-optimal solutions within 900-1000 iterations while continuing to make incremental improvements through the remaining iterations.



(a)



(b)

Figure 7: Convergence characteristics of the SA_EVPS algorithm for the 1410-bar dome optimization: (a) Convergence history for 20 independent runs over 1000 iterations, (b) Detailed view of the final convergence phase (iterations 900-1000) showing algorithmic stability

The optimization process employed a population size of 55 particles with a maximum of 1000 structural analyses per run. This configuration was specifically tuned for the double-layer structure's complexity, balancing computational efficiency with solution quality. The experiment was repeated 20 times to ensure statistical validity and demonstrate algorithmic consistency.

The SA_EVPS algorithm's superior performance on this benchmark problem demonstrates its readiness for practical application in large-scale structural engineering projects. The combination of optimal weight reduction, precise constraint satisfaction, and algorithmic robustness establishes SA_EVPS as a highly effective tool for complex structural optimization challenges in modern engineering practice.

4. CONCLUSIONS

The Self-Adaptive Enhanced Vibrating Particle System (SA_EVPS) algorithm demonstrates superior performance for large-scale dome truss optimization with frequency constraints. Through comprehensive evaluation on three benchmark structures, the following key conclusions are established:

1. **Optimization Performance**: SA_EVPS consistently achieved the best solutions across all test cases - 6065.72 kg (600-bar), 37492.63 kg (1180-bar), and 10310.25 kg (1410-bar dome) - representing improvements of 0.03-0.9% over existing methods. While percentage improvements appear modest, they translate to significant material savings in practical applications.
2. **Algorithmic Reliability**: The standard deviations of 11.26 kg, 48.37 kg, and 55.95 kg for the three structures respectively demonstrate exceptional consistency, with 52-92% reduction in variability compared to traditional algorithms. This reliability is crucial for practical engineering implementation.
3. **Constraint Handling**: All frequency constraints were satisfied within 0.01-0.06% of specified limits, demonstrating precise constraint management essential for resonance avoidance in structural design.
4. **Self-Adaptive Capability**: The elimination of manual parameter tuning through dynamic adaptation of damping coefficients and weight parameters enables robust performance without requiring optimization expertise, making the algorithm accessible to practicing engineers.

The SA_EVPS algorithm represents a practical advancement in structural optimization, providing a parameter-free, reliable tool for frequency-constrained dome design. Future work should explore multi-objective formulations and integration with surrogate modeling for enhanced computational efficiency in very large-scale applications.

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