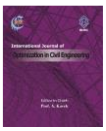
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# AN EFFICIENT MATHEMATICAL MODEL AND STEP-BY-STEP SOLUTION METHOD FOR THE CAPACITATED VEHICLE ROUTING PROBLEM VIA CUSTOMER MARKING

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## ABSTRACT

This study investigates the delivery vehicle routing problem (VRP) in urban areas, where the demands of geographically dispersed customers must be fulfilled by a heterogeneous fleet of capacity-constrained vehicles while minimizing total travel distance. VRP is a well-known and highly complex combinatorial optimization problem, for which diverse solution methods have been proposed. In conventional routing formulations, binary variables are typically defined for every customer pair to determine traversals, causing model dimensions to grow quadratically with the number of customers. To overcome this limitation, this paper introduces a novel mathematical formulation that drastically reduces the number of binary variables by sequencing visited customers along selected routes. Furthermore, to efficiently handle large-scale instances, an iterative, step-by-step solution method based on sequential customer marking is developed. Extensive implementations on various instances demonstrate the high computational efficiency and practical applicability of the proposed approach.

**Keywords:** Vehicle Routing Problem; Urban delivery; Customer marking; Mathematical modeling; Step-by-step solution method.

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## 1. INTRODUCTION

With the increasing penetration of the internet and the digitalization of the economy, the pattern of goods purchasing is shifting extensively from traditional brick-and-mortar stores to online shopping. This transformation brings significant benefits to merchants by reducing

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operational costs such as rent, broadening market access, and increasing sales potential, and to buyers by saving time, providing easier access to diverse goods, and lowering transportation costs [1]. However, the rapid growth of e-commerce has exacerbated logistical challenges, particularly in the final-delivery stage. One of the key challenges is optimal planning and efficient routing of the delivery fleet to strike a balance between timely delivery, which ensures customer satisfaction, and the minimization of operational and traffic costs. In this context, utilizing vehicle capacities effectively and designing short, integrated routes to serve a larger number of customers per trip can simultaneously promote both efficiency and sustainability in urban delivery [2].

The Vehicle Routing Problem (VRP) is one of the fundamental pillars of network optimization. Generally, VRP involves one or more vehicles departing from a distribution center to service all or a subset of customers, and subsequently returning to the depot or a specified destination, such that transportation costs, delivery times, or other objective metrics are minimized. A successful solution to the VRP directly translates into reduced fuel and labor costs, an increased number of daily deliveries, minimized delays, and lower carbon emissions. Conversely, its primary challenges include urban traffic congestion, customer dispersion, time windows, the demand for rapid services including same-day or hourly delivery, and escalating customer expectations. Graph theory has been widely applied in various engineering disciplines to model and analyze complex network structures [3]. Similarly, the VRP can be formulated as a graph-based optimization problem, where mathematical modeling provides a systematic framework for representing the problem, incorporating practical constraints, and developing more efficient solution approaches.

One of the most classic and well-known variants of this problem is the Traveling Salesperson Problem (TSP), where a single salesperson must visit all customers in a complete graph along the shortest route and return to the starting point, determining the optimal visiting sequence to minimize total travel distance. Various extended variants of the problem have been developed in the literature, including the Capacitated Vehicle Routing Problem (CVRP) where vehicle payload capacities are restricted [4], the Vehicle Routing Problem with Time Windows (VRPTW) where customer demands must be fulfilled within predefined time intervals [5], the Open Vehicle Routing Problem (OVRP) which relaxes the requirement for vehicles to return to the depot [6], the Vehicle Routing Problem with Backhauls (VRPB) which accommodates both delivery and pickup of returned goods from customers [7], the Split Delivery Vehicle Routing Problem (SDVRP) allowing a customer's demand to be fulfilled by multiple vehicles [8], and the Mixed Fleet Vehicle Routing Problem (MFVRP) which utilizes diverse vehicles with varying capacities and characteristics [9].

From a computational complexity perspective, these problems belong to the class of NP-hard problems [8], prompting the widespread use of heuristic and metaheuristic approaches instead of exact solution methods [10]. Although various mathematical models predominantly formulated as Mixed-Integer Linear Programming (MILP) and exact solution methods such as branch-and-cut [11], branch-and-price [12], and column generation [13] have been proposed, tackling large-scale and real-world instances typically relies on fast, non-exact methods. Prominent metaheuristic algorithms applied in this domain include the Ant Colony Optimization [14], Genetic Algorithm [15], Particle Swarm Optimization [16], and Tabu Search algorithms [17].

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Across all aforementioned variants, a critical variable significantly impacting model size and computational run-time is the binary decision variable determining whether a traversal occurs between each customer pair i and j. Defining such binary variables for every network arc results in models with extremely large dimensions scaling quadratically with the number of customers, thereby introducing severe computational bottlenecks, particularly for real-world instances. To overcome this challenge, this research proposes an innovative formulation alongside a step-by-step solution method, aiming to drastically reduce model dimensions and enhance computational efficiency.

## 2. PROBLEM DEFINITION, MATHEMATICAL MODELING, AND SOLUTION APPROACH

In this problem, a company or distribution center records customer demands and addresses, utilizing a fleet of capacity-constrained vehicles to deliver goods. Different vehicle types are considered based on payload capacities, and each vehicle can fulfill the orders of a subset of customers sequentially. The objective is to determine appropriate delivery routes for these diverse vehicles such that all customer demands are satisfied along minimized routes.

Due to the high computational complexity of simultaneously determining all routes within a single monolithic model, the routes are determined in an iterative, step-by-step manner. In this approach, during each iteration, a specific customer is designated as a reference point based on a predefined priority, and a route is sought that satisfies the following features:

1. The reference point must be visited in the selected route;
2. The payload capacity of the chosen vehicle type is utilized as efficiently as possible;
3. The maximum distance between any customer on the route and the reference point is minimized;
4. The total travel distance is minimized.

Consequently, in each iteration, by solving a Mixed-Integer Linear Programming (MILP) model, a route comprising several specific customers is identified, and these customers are removed from the search set for subsequent iterations. This process continues until all customers are visited. The priority rules for selecting reference points can vary, such as choosing the customer farthest from the depot, the customer with the largest or highest-value demand, the customer with the earliest order placement, or selecting customers randomly.

To formulate the problem as an MILP model, the sets $I$, $L$, and $K$ are defined to represent the set of customer points, the visiting sequence or order of customers in a route, and the vehicle types, respectively. We assume $|K|$ vehicle types are available, where the capacity of each type is denoted by $cap_k$, and a binary variable $v_k$ determines which vehicle type is deployed for the current trip. Customer indices are denoted by $i$, and a binary variable $y_i$ is defined to determine the visited customers in the current route/trip. It is also assumed that a maximum of $|L|$ customers are visited per route, indexed sequentially from 1 to $|L|$. The index $l$ represents the visiting turn of a customer along the route, and the binary variable $z_{il}$ determines which customer is visited at turn $l$ on the selected route. In conventional routing

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formulations, a binary variable of the form $x_{ij}$ is typically defined to indicate whether a traversal occurs from node $i$ to node $j$. The number of such variables grows quadratically with the number of customers. In contrast, by defining the two aforementioned variable types ($y_i$ and $z_{il}$), the number of variables scales linearly with the number of customers. For illustration, consider a problem with 100 customer locations. A conventional formulation based on ordered pairs of locations may require 100 100×100=10000 binary variables, whereas the proposed formulation would require 100×10=1000 such variables if each vehicle can visit at most 10 customers. Under these assumptions, the proposed formulation reduces the number of routing-related binary variables by 90%.

Based on the problem description and the defined decision variables, the MILP model for finding a route containing the reference point for the Capacitated Vehicle Routing Problem is presented as follows:

$$Min\ z = c_1 Re + c_2 Q + c_3\left(\sum_{i\in I} To_i + \sum_{i\in I}\sum_{j\in I} T_{ij}\right) \tag{1}$$

Subject to:

$$\sum_{l\in L} z_{il} \le 1, \qquad \forall i \in I \tag{2}$$

$$\sum_{i\in I} z_{il} \le 1, \qquad \forall l \in L \tag{3}$$

$$y_i = \sum_{l\in L} z_{il} \qquad \forall i \in I \tag{4}$$

$$\sum_{i\in I} z_{il+1} \le \sum_{i\in I} z_{il}, \qquad \forall l \in L, l < |L| \tag{5}$$

$$y_i + y_j \le 1, \qquad \forall i,j \in I, t_{ij} > T_{max} \tag{6}$$

$$\sum_{l\in L} z_{il} = 1, \qquad i \in I, i = \text{reference point} \tag{7}$$

$$\sum_{i\in I} d_i y_i = \sum_{k\in K} cap_k v_k - Re, \tag{8}$$

$$\sum_{k\in K} v_k = 1, \tag{9}$$

$$To_i \ge to_i(z_{il}), \qquad \forall i \in I, l = 1 \tag{10}$$

$$T_{ij} \ge t_{ij}\left(z_{il} + z_{jl+1} - 1\right), \qquad \forall i,j \in I, \forall\, l \in L, l < |L| \tag{11}$$

$$\sum_{i\in I} To_i + \sum_{i\in I}\sum_{j\in I} T_{ij} \ge to_i y_i, \quad \forall i \in I \tag{12}$$

$$Q \ge td_i y_i, \quad \forall i \in I \tag{13}$$

$$Re, Q, To_i, T_{ij} \ge 0, \quad z_{il}, y_i, v_k \in \{0,1\} \tag{14}$$

In the objective function (1), three main metrics are minimized: the total unused capacity of the dispatched vehicles ($Re$), the maximum distance between any customer on the route and the reference point ($Q$), and the total travel distance. Constraints (2) and (3) guarantee that each customer is visited at most once, and at most one customer is assigned to each

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visiting position. Constraint (4) links the binary decision variable $y_i$ with the visiting sequence, ensuring that if a customer is included in the route, they must be visited during one of the trip stages. Constraint (5) maintains sequence integrity by preventing any gaps between visiting turns. According to constraint (6), if the distance between two customers exceeds a predefined threshold $T_{max}$, they cannot be assigned to the same trip. The designation of the reference customer is enforced by constraint (7). Furthermore, constraint (8) satisfies customer demands ($d_i$) while respecting vehicle payload capacity limits, and constraint (9) governs the selection of the appropriate vehicle type. The distance from the depot to each customer and the pairwise distance between customers are denoted by $to_i$ and $t_{ij}$, respectively, whereas the travel distance from the depot to the first customer ($To_i$) and between consecutive customers ($T_{ij}$) are modeled via constraints (10) and (11). Constraint (12) acts as an auxiliary constraint ensuring that if a customer is served on the route, the route length is at least equal to the distance from that point to the depot. Finally, constraint (13) bounds the maximum distance between any customer on the selected route and the reference point ($td_i$), and constraint (14) defines the variable domains.

## 3. IMPLEMENTATION AND RESULTS ANALYSIS

To evaluate the proposed model and solution method, a set of random test instances was generated. Initially, a 40×40 two-dimensional coordinate region was defined, within which the depot and 100 customers were randomly distributed. Since the problem is primarily oriented toward urban logistics, Manhattan distance was employed to compute the distance between any two coordinate points $a = (a_1, a_2)$ and $b = (b_1, b_2)$ as $distance(a, b) = | a_1 - b_1 | + | a_2 - b_2 |$. Customer demands were randomly generated from the discrete uniform set $[1,20]$. Two types of vehicles with capacities of 15 and 30 units were considered, without any upper limit on the available fleet size. Based on these specifications, a limit of 7 visits per trip has been set. The mathematical models were implemented in the AIMMS optimization software environment and solved using the CPLEX 12 solver on a personal computer equipped with a 2 GHz CPU and 2 GB of RAM.

### *3.1. Implementation on random datasets with 100 customers*

First, three distinct random problem instances were generated based on the aforementioned specifications, and the proposed algorithm and MILP model were applied to them. Three different strategies were investigated for selecting the reference customer: choosing the customer furthest from the depot, selecting the customer with the highest demand, and choosing a customer randomly. The corresponding results are summarized in Table 1.

As indicated in Table 1, no single reference customer selection strategy consistently outperforms the others in terms of total travel distance. Across the three test instances, each priority rule achieved the best performance in total travel distance once, and the performance differences among them are relatively minor. However, regarding total deployed vehicle capacity and unused capacity, the strategy of selecting the furthest point consistently performed better than the other two approaches across all three instances. Furthermore, the

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solution quality metrics including the number of customers per route, the maximum distance from the reference point, and the execution time remain closely aligned, with no substantial deviations observed.

The number of algorithmic iterations, which corresponds to the number of solved MILP models, equals the total number of trips and consequently the number of vehicles utilized, as reported in column 5 of Table 1. A noteworthy observation is that the total execution time of the algorithm is approximately 1 minute, which is highly acceptable for a problem instance involving 100 customers.

Table 1: Solution report assuming 100 customers and two vehicle types with capacities of 15 and 30 units.

| Case | Reference point priority | Total distance | Gap to best (%) | Number of vehicles: type 1 + type 2 | Total deployed capacity | Unused capacity | Min-Max customers per route | Max distance to reference point | Execution time (min) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Furthest | **887** | **0** | 22+8=30 | **570** | **(%1.05)6** | 1-6 | 20 | 1.01 |
| | Demand | 949 | 6.9 | 21+9=30 | 585 | (%3.5)21 | 1-7 | 26 | 1.03 |
| | Random | 891 | 0.45 | 22+9=31 | 600 | (%6)36 | 1-5 | 18 | 1 |
| 2 | Furthest | 831 | 3.6 | 17+8=25 | **495** | **(%1.61)8** | 1-7 | 24 | 1.02 |
| | Demand | **802** | **0** | 22+6=28 | 510 | (%4.5)23 | 1-6 | 25 | 1.08 |
| | Random | 823 | 2.6 | 24+5=29 | 510 | (%4.5)23 | 1-6 | 28 | 0.99 |
| 3 | Furthest | 917 | 3.1 | 23+5=28 | **495** | **(%1.81)9** | 1-6 | 28 | 1.03 |
| | Demand | 903 | 1.5 | 28+4=32 | 540 | (%10)54 | 1-7 | 28 | 1.01 |
| | Random | **889** | **0** | 23+5=28 | 495 | (%1.81)9 | 1-6 | 30 | 1.04 |

The delivery routes obtained by applying the method to test instance 1 (where the depot is centrally located in the urban area) under the three reference customer selection strategies are illustrated in Figures 1 to 3. In these plots, the initial travel leg from the depot to the first customer of each route is omitted for clarity. In Figure 1, the red points representing the reference points across various iterations are predominantly distributed at greater distances from the depot, whereas in the subsequent two figures, these points exhibit a more uniform spatial dispersion across the coordinate plane.

### *3.2. Evaluation of increased vehicle capacities*

One of the key observations during the computational experiments was that increasing vehicle capacities leads to a higher number of customers visited per route, thereby reducing both the total number of deployed vehicles and the overall travel distance. This occurs because when two routes merge, the intermediate travel segment from the depot to one of the previously visited points is eliminated.

Conversely, expanding the payload capacity of a vehicle increases the likelihood of leaving a portion of its capacity unutilized. Because the capacity threshold is higher, serving more customers is required to fully load the vehicle; however, since route travel distances are bounded by upper limits, it is not always feasible to incorporate additional customers, resulting in unused capacity slack. Another important observation is that the execution time increases significantly due to the expansion of the solution space. These findings are clearly demonstrated in Table 2.

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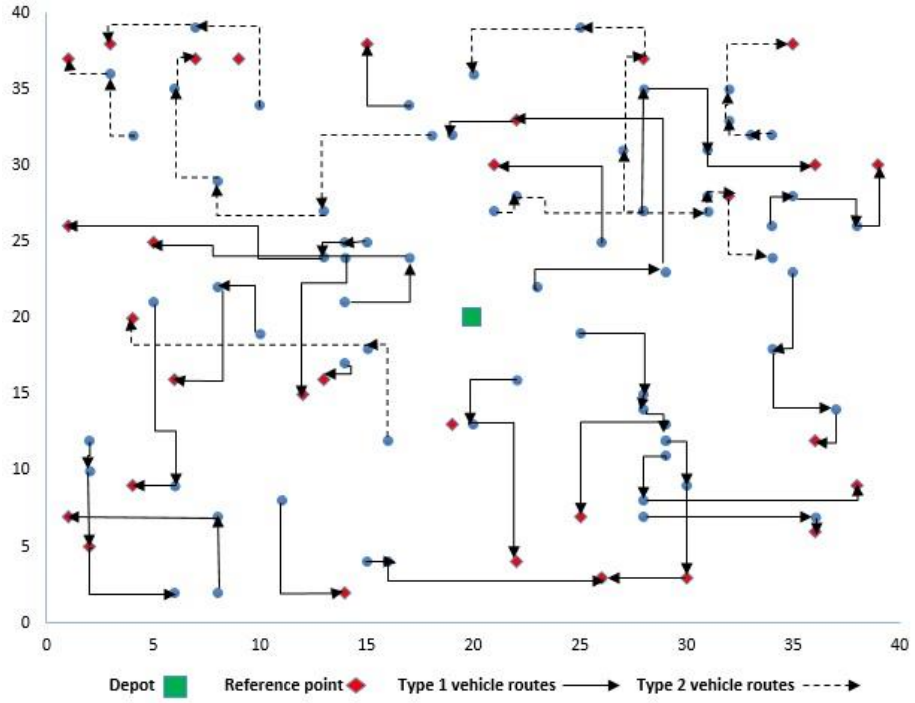



Figure 1: Obtained delivery routes for test instance 1 with the furthest point as the reference point.

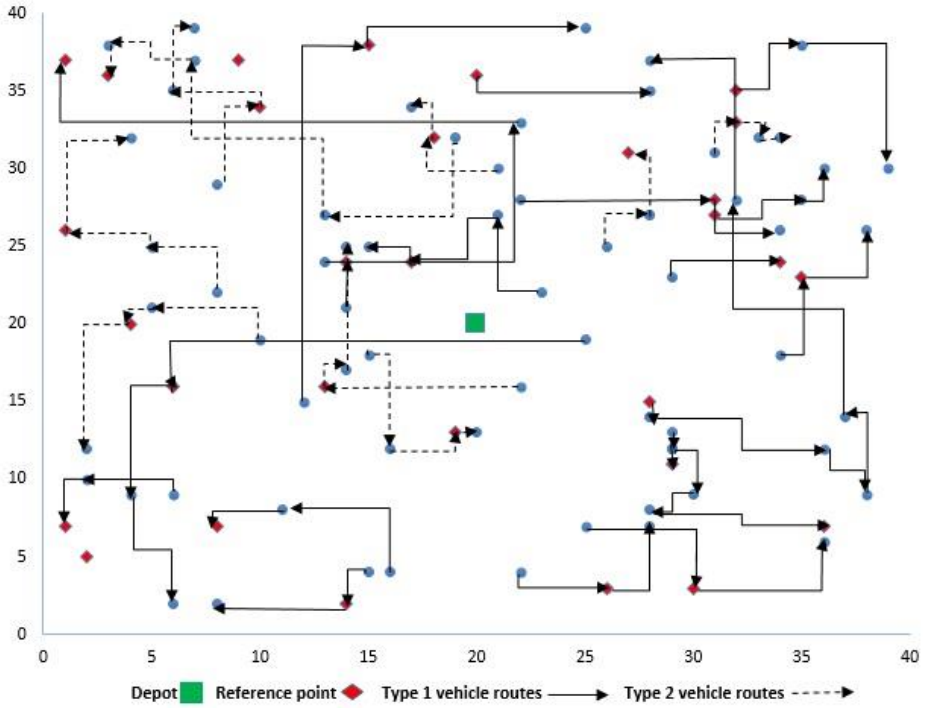



Figure 2: Obtained delivery routes for test instance 1 with the maximum demand point as the reference point.

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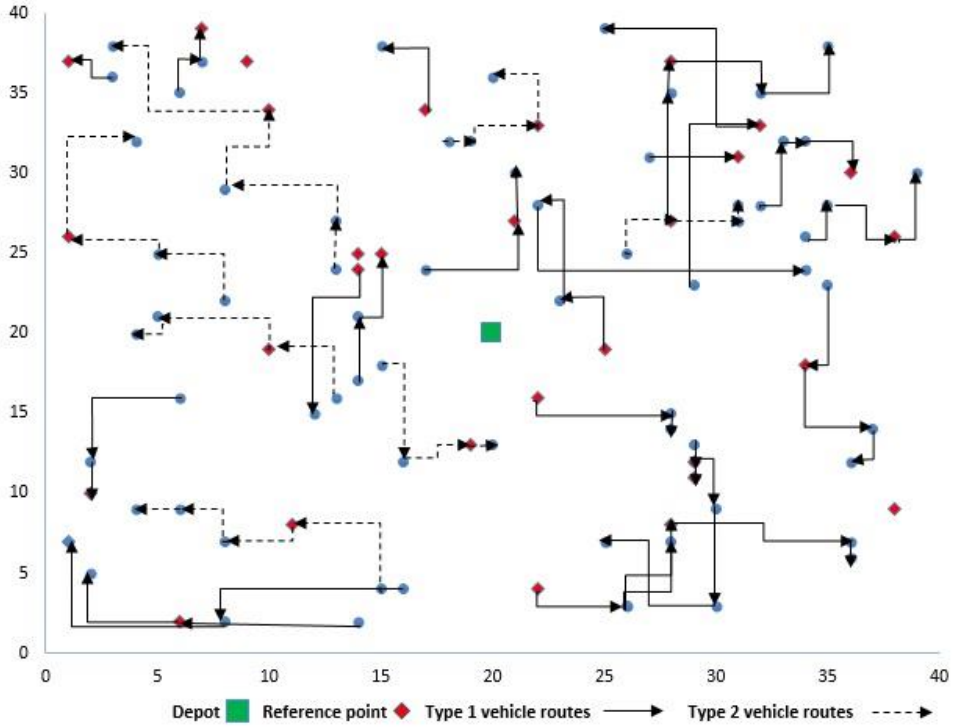



Figure 3: Obtained delivery routes for test instance 1 using the random selection strategy as the reference point.

Table 2: Impact of increasing vehicle capacities on solution performance.

| Reference priority | Capacity of two types of vehicles | Total distance | Gap to best (%) | Number of vehicles: type 1 + type 2 | Total deployed capacity | Unused capacity | Min-Max customers per route | Mean customers per route | Max distance to reference Point | Execution time (min) |
|---|---|---|---|---|---|---|---|---|---|---|
| Furthest | (15,30) | 917 | 13 | 23+5=28 | 495 | **9(1.81%)** | 1-6 | 3.57 | 28 | **1.03** |
| | (30,60) | **811** | 0 | 20+0=20 | 600 | 114(19%) | 3-7 | 5 | 26 | 4.21 |
| Demand | (15,30) | 903 | 30 | 28+4=32 | 540 | **54(10%)** | 1-7 | 3.12 | 28 | **1.01** |
| | (30,60) | **693** | 0 | 19+0=19 | 570 | 84(14.7%) | 3-7 | 5.26 | 25 | 4.14 |
| Random | (15,30) | 957 | 20 | 27+4=31 | 525 | **9(1.7%)** | 1-6 | 3.22 | 28 | **1.03** |
| | (30,60) | **791** | 0 | 19+0=19 | 570 | 84(14.7%) | 2-7 | 5.26 | 30 | 4.98 |

### *3.3. Impact of increasing customer scale on algorithm performance*

Another critical factor influencing algorithm execution time and solution quality is the scale of customer demand points. In the subsequent experiments, while keeping the vehicle capacity limits constant, the total number of customers was doubled, with customer demands similarly generated from the discrete uniform set [1,20]. The primary findings indicate that when the number of customers increases under fixed vehicle capacities, the average number

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of customers visited per route remains relatively stable, while the total number of trips and the overall travel distance approximately double. Due to the increased fleet size, the cumulative transport capacity and the associated capacity slack also increase. The execution time of the MILP model in each iteration remains largely unaffected because the number of customers per individual route does not change drastically; however, because the total customer count is doubled, the frequency of model execution and the overall algorithm runtime increase proportionally. These outcomes are summarized in Table 3.

Table 3: Impact of increasing the number of customers on solution performance.

| Reference priority | (\|I\|,\|L\|) | Total distance | Number of vehicles: type 1 + type 2 | Total Deployed Capacity | Unused Capacity | Min-Max Customers per Route | Mean Customers per Route | Max Distance to Reference Point | Execution Time (min) |
|---|---|---|---|---|---|---|---|---|---|
| Furthest | (100,7) | 917 | 23+5=28 | 495 | 9(1.81%) | 1-6 | 3.57 | 28 | 1.03 |
| | (200,7) | 1551 | 45+10=55 | 975 | 5(0.5%) | 2-7 | 3.36 | 17 | 2.16 |
| Demand | (100,7) | 903 | 28+4=32 | 540 | 45(10%) | 1-7 | 3.12 | 28 | 1.01 |
| | (200,7) | 1545 | 46+11=57 | 1020 | 50(4.9%) | 2-7 | 3.5 | 26 | 2.87 |
| Random | (100,7) | 957 | 27+4=31 | 525 | 9(1.7%) | 1-6 | 3.22 | 28 | 1.03 |
| | (200,7) | 1528 | 44+11=55 | 990 | 20(2.02%) | 1-7 | 3.63 | 30 | 2.22 |

## 4. CONCLUSIONS

This study introduced a novel formulation and solution framework for the classical VRP, reducing the number of binary variables by sequencing customers along vehicle routes rather than defining customer-pair variables. An iterative algorithm based on selective customer marking was also developed to efficiently solve large-scale instances while balancing solution quality and computational time. Numerical experiments showed that the algorithm solves instances with 100 customers in approximately one minute. While the three reference point selection criteria (maximum distance, highest demand, and random selection) resulted in similar total travel distances, selecting the furthest point improved vehicle capacity utilization. Increasing vehicle capacity reduced the number of trips and overall travel distance but could increase computational complexity. Furthermore, execution time increased approximately linearly with the number of customers, whereas larger vehicle capacities and the corresponding increase in the maximum number of visits per trip introduced additional computational challenges. Despite not guaranteeing global optimality, the proposed approach maintained relatively stable performance across different scenarios and offered a practical balance between solution quality, computational efficiency, and implementation feasibility. Overall, it decomposes a computationally demanding VRP into manageable subproblems, making it a promising approach for urban logistics, goods distribution, and supply chain management. Future work will explore: (1) integrating advanced machine learning and neural network approaches to solve very large-scale

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instances based on [18], (2) incorporating stochastic travel times and dynamic customer requests, and (3) applying the model to real-world multi-depot urban distribution networks.

## REFERENCES


1. Kitjacharoenchai P, Ventresca M, Moshref-Javadi M, Lee S, Tanchoco JM, Brunese PA. Multiple traveling salesman problem with drones: mathematical model and heuristic approach. *Comput Ind Eng*. 2019;**129**:14–30. doi:10.1016/j.cie.2019.01.020.
2. Boysen N, Fedtke S, Schwerdfeger S. Last-mile delivery concepts: a survey from an operational research perspective. *OR Spectr*. 2021;**43**(1):1–58. doi:10.1007/s00291-020-00607-8.
3. Kaveh A. *Structural Mechanics: Graph and Matrix Methods*. 3rd ed. Research Studies Press (John Wiley); 2004.
4. Landete M, Leal M, Monge JF, Rodríguez-Martín I. The capacitated vehicle routing problem with planned downtime. *Comput Optim Appl*. 2025;**92**(2):655–81. doi:10.1007/s10589-025-00713-9.
5. Campos R, Coelho LC, Munari P. New formulations for the robust vehicle routing problem with time windows under demand and travel time uncertainty. *OR Spectr*. 2025;**47**(2):411–53. doi:10.1007/s00291-024-00781-z.
6. Soares VC, Roboredo M. On the exact solution of the multi-depot open vehicle routing problem. *Optim Lett*. 2024;**18**(4):1053–69. doi:10.1007/s11590-023-02072-y.
7. Sreerag RS, Shanmugam PV. Vehicle routing model with backhauling for distribution of fresh vegetables among small retail channels: a case study in Kerala, India. *OPSEARCH*. 2025;**12**:1–21. doi:10.1007/s12597-025-00909-x.
8. Archetti C, Feillet D, Gendreau M, Speranza MG. Complexity of the VRP and SDVRP. *Transp Res Part C Emerg Technol*. 2011;**19**(5):741–50. doi:10.1016/j.trc.2009.12.006.
9. Wang R, Zhang F, Ding L, Jiang J, Han Z. A novel approach for optimizing mixed hazardous material fleet vehicle routing problem method through intermediate bulk container sharing with adaptive intuitionistic fuzzy large neighborhood search. *Int J Fuzzy Syst*. 2025;**27**(6):1827–45. doi:10.1007/s40815-024-01875-0.
10. Kaveh A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*. 3rd ed. Springer; 2014. doi:10.1007/978-3-030-59392-6.
11. Manousakis E, Repoussis P, Zachariadis E, Tarantilis C. Improved branch-and-cut for the inventory routing problem based on a two-commodity flow formulation. *Eur J Oper Res*. 2021;**290**(3):870–85. doi:10.1016/j.ejor.2020.08.047.
12. Desaulniers G, Rakke JG, Coelho LC. A branch-price-and-cut algorithm for the inventory-routing problem. *Transp Sci*. 2016;**50**(3):1060–76. doi:10.1287/trsc.2015.0635.
13. Xu K, Shen L, Liu L. Enhancing column generation by reinforcement learning-based hyper-heuristic for vehicle routing and scheduling problems. *Comput Ind Eng*. 2025;**206**:111138. doi:10.1016/j.cie.2025.111138.
14. Suppan M, Hanne T, Dornberger R. Ant colony optimization to solve the rescue problem as a vehicle routing problem with hard time windows. In: *Proc Int Joint Conf*


[ Downloaded from ijoce.iust.ac.ir on 2026-11-09 ]

[ DOI: 10.22068/ijoce.2026.16.4.691 ]

*Adv Comput Intell (IJCACI 2021)*. Springer Nature Singapore; 2022:53–65. doi:10.1007/978-981-19-0332-8_5.
15. Euchi J, Sadok A. Hybrid genetic-sweep algorithm to solve the vehicle routing problem with drones. *Phys Commun*. 2021;**44**:101236. doi:10.1016/j.phycom.2020.101236.
16. Marinakis Y, Marinaki M, Migdalas A. Particle swarm optimization for the vehicle routing problem: a survey and a comparative analysis. In: *Handbook of Heuristics*. Springer Nature Switzerland; 2025:1563–96. doi:10.1007/978-3-032-00385-0_42.
17. Archetti C, Boland N, Speranza MG. A matheuristic for the multivehicle inventory routing problem. *INFORMS J Comput*. 2017;**29**(3):377–87. doi:10.1287/ijoc.2016.0737.
18. Kaveh A. *Applications of Artificial Neural Networks and Machine Learning in Civil Engineering*. Springer; 2024. doi:10.1007/978-3-031-66051-1.

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