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# GENERATION OF ENDURANCE TIME ACCELERATION FUNCTIONS USING THE WAVELET TRANSFORM

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## ABSTRACT

Endurance Time Acceleration Functions are specially predesigned intensifying excitation functions that their amplitude increases with time. On the other hand, wavelet transform is a mathematical tool that indicates time variations of frequency in a signal. In this paper, an approach is presented for generating endurance time acceleration functions (ETAFs) whose response spectrum is compatible with the European Code regulations (EC8) elastic spectrum. Method applied is a modification of data in time and frequency domain. For this purpose, wavelet transform has been used to decompose a series of random points to several levels such that each level covers a special range of frequency, then every level is divided into the numbers of equal time intervals and each interval of time is multiplied by a variable. Subsequently, the mathematical unconstrained optimization algorithm is used to calculate the variables and minimize error between response and target spectra. The prosed procedure is used in two methods. Then with two methods, two different acceleration functions are produced.

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KEY WORDS: endurance time acceleration functions; response spectrum; wavelet transform; unconstrained optimization algorithm

## 1. INTRODUCTION

One of the major objectives in the seismic design codes is providing appropriate safety margin

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against the structural failure and inconstancy caused the destructive earthquakes. Among the existing shortcomings of conventional design methods and the guidelines is the impossibility to evaluate explicitly the performance of structures and components. Thus, structural codes and design guidelines are moving towards adoption of performance-based design approaches. The aim of a performance-based design is to built structures such that its performance against different earthquakes is predictable [1,2]. Equivalent static analysis, modal analysis and dynamic time-history analysis are all common and simple methods utilized in the seismic analysis and design of structures. Existing shortages in the mentioned methods and developments in the high speed and accurate computational tools lead researchers to correcting old and approximate methods. In this context, performance based design methods such as non-linear static, capacity spectrum, time history and incremental dynamic analysis (IDA) were introduced [3,4].

Since 2004, the Endurance Time (ET) method has been introduced for the performance-based seismic analysis and design of structures. In this method, the computational demand is considerably reduced by subjecting the structure to an intensifying acceleration function (AF) and monitoring the objective performance indices through time. Generating efficient dynamic function is essential for success of the ET method [5].

Estekanchi et al. [6] explained the concept, criteria and generation of the first series of endurance time acceleration functions (ETAFs). Estekanchi et al. [7] generating code compliant uniformly intensifying ETAFs has been explained. Nozari and Estekanchi [8] developed optimization procedure of ETAFs by modification objective function from the continuous time condition into the discrete time state. In these papers, the target spectrum is design spectrum of Iranian National Building Code (INBC).

The newly developed wavelet analysis has emerged as a powerful tool to analyze temporal variations in frequency content. Recent applications of the wavelet transform to engineering problems can be found in several studies that refer to dynamic analysis of structures, damage detection, system identification, etc [9]. Newland [10] applied wavelets for analyzing vibration signals, and developed special wavelets and techniques for engineering purpose. Rajasekaran et al. [11], and Ghodrati Amiri et al. [12] developed the wavelet analysis for generating earthquake accelerograms.

Estekanchi et al for generation of ETAFs used only time domain modification procedure and this procedure considered the high volume of computations and is a time consuming process. In this study, a new method for generation of ETAFs is presented in time and frequency domain optimization procedures. For modification of the data in frequency domain wavelet transform is utilized. Also, for modification of the data in time domain unconstrained optimization procedure is used. The proposed procedure is employed by two methods; the number of optimization variables is constant in Method I and changes in Method II in each iteration. Different template spectrums such as the design spectrum of Iranian National Building Code (INBC), ASCE design spectrum and average of response spectra from sets of ground motions can be considered. In this study, the EC8 [13] design spectrum is considered as the target response spectrum. The comparisons with the other available ETAFs indicants a considerable reduction of the computational time.

## 2. GENERATION OF ENDURANCE TIME ACCELERATION FUNCTIONS

In the first generation of ETAFs, the procedure of generating the acceleration functions started from a random vibration accelerogram similar to a white noise which was modified by a filter in the frequency domain and then made compliant with a typical code design response spectrum. The resulting stationary accelerogram was then modified by applying a linear profile function that made it intensify with respect to peak accelerations at different time intervals. These accelerograms served well the purpose of demonstrating the concept of ET analysis, but could not be expected to result in quantitatively significant results [5].

In this paper, the second generation of ETAFs is generated. In the second generation of the ETAFs, in order the ETAFs to correspond to the average code compliant design level earthquakes, the concept of the response spectrum is more directly involved. These ET acceleration functions are designed in such a way that it produces dynamic responses equal to the code's design spectrum at a particular time, $t_{Target}$, and objective response in all other times is defined by a linear function of time based on the target response as follows:

$$s_{aT}(T_i, t_j) = \frac{t_j}{t_{Target}} s_{ac}(T_i) \tag{1}$$

Where, $T$ is the fundamental period of the structure, $t_{Target}$ is the target time, $S_{aT}$ is the spectral acceleration response of structure and $S_{aC}$ is the codified spectral acceleration. In this ETAFs template spectrum is the design spectrum of EC8 for soil type B. The elastic displacement response spectrum for oscillators with 5% ratio of critical damping and natural period T, is defined by the European seismic code provisions (EC8) [13] by the following relationships:

$$S_a(T) = \begin{cases} 1.2a_g\left(1 + \frac{1.5T}{0.15}\right) & T < 0.15 \\ 3a_g & 0.15 \le T < 0.5 \\ \frac{1.5a_g}{T} & 0.5 \le T < 2 \\ \frac{3a_g}{T^2} & 2 \le T < 5 \\ \frac{1.2a_g}{T^2}\left[2.5 - 1.5\frac{T-5}{10-5}\right] & 5 \le T < 10 \\ \frac{1.2a_g}{T^2} & 10 \le T \end{cases} \tag{2}$$

In this equation, $a_g$ is the PGA, and it is equal to 35% of the acceleration of the gravity (g).

For calculation of the time history responses due to a dynamic input, we can consider the differential equation of motion for a SDOF system under an earthquake excitation:

$$\ddot{u}(t) + 2\xi w_n \dot{u}(t) + w_n^2 u(t) = -\ddot{u}_g(t) \tag{3}$$

Where, $x$ is the damping ratio, $w_n$ is the natural circular frequency and $\ddot{u}_g(t)$ is the ground excitation time history. The acceleration response function can be calculated from the absolute acceleration responses as follows:

$$S_{i,j}^{a} = S_a(T_i, t_j) = \max\left(\left|\ddot{u}(t) + \ddot{u}_g(t)\right|\right) \qquad 0 \le t \le t_j \tag{4}$$

Further, the displacement response function can be obtained from the relative displacement responses as:

$$S_{i,j}^{u} = S_u(T_i, t_j) = \max\left(\left|u(t)\right|\right) \qquad 0 \le t \le t_j \tag{5}$$

In conventional methods the acceleration function, with its response spectrum being as relationships (1), is formulated as an unconstrained optimization problem in the time domain with the following objective function:

$$F_{abs}(a_g) = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left[s_{i,j}^{a} - s_{i,j}^{aT}\right]^2 + a\left[s_{i,j}^{u} - s_{i,j}^{uT}\right]^2\right)}{n \times m}} \tag{6}$$

Where $\alpha_g$ is the acceleration function, *n* and *m* are the number of period and time points in the optimization process. This function is also known as the base error. To calculate the base error, period points between 0 to 10 seconds with uniform distribution with a step of 0.02s, and all of the time steps are utilized. $a$ is an optimization weighting factor considered as 1.

In conventional method, since the optimization process is performed in the time domain, $\alpha_g$ is as considered as the optimization variable. For example, for generating an acceleration function, if the duration of the AF is equal to 20.47 seconds consisting of 2048 acceleration points in 0.01s time steps, the number of variables of the optimization will be 2048. Thus, this optimization process is a time consuming process due to considering a high number of the variables and also due to the objective function count. e.g. to produce each AF of ETA20d series, more than 120 hours is required by utilizing Pentium IV CPU with a frequency of 2800 GHz [8].

## 3. WAVELET AND WAVELET PACKET TRANSFORM

### *3.1. Basis function*

Fast Fourier transform (FFT) is an excellent tool for finding the frequency components of a signal. A disadvantage of the FFT is that the frequency components can only be extracted from the complete duration of a signal. The frequency components are obtained by an averaging over the entire length of the signal. Therefore, it is not a suitable tool for a non-stationary signal such as the impulse response of cracked beams, vibration generated by faults in a gearbox, and structural response to wind storms, just to name a few. These types of problems associated with FFT can be resolved by using wavelet analysis. It provides a powerful tool to characterize local features of a signal. Unlike Fourier transform, where the function used as the basis of decomposition is always a sinusoidal wave, other basis functions can be selected for wavelet shape according to the features of the signal. The basis function in wavelet analysis is defined by two parameters named scale and translation. This property leads to a multi-resolution representation for non-stationary signals. As mentioned before, a basis function (or mother wavelet) is used in wavelet analysis. For a wavelet of order $N$, the basis function can be represented as:

$$y(n) = \sum_{j=0}^{N-1} (-1)^n c_j (2n + j - N + 1) \tag{7}$$

Where $c_j$ is the $j$th coefficient. The basis function should satisfy the following two conditions (Eqs. (8) and (9)):

The basis function integrates to zero, i.e.

$$\int_{-\infty}^{+\infty} y(t)dt = 0 \tag{8}$$

It is square integrable or, equivalently, has finite energy, i.e.

$$\int_{-\infty}^{+\infty} |y(t)|^2 dt < \infty \tag{9}$$

Equation (8) suggests that the basis function be oscillatory or have a wavy shape. Equation (9) implies that most of the energy in the basis function is confined to a finite duration. The important properties of basic functions are ‘orthogonality’ and ‘biorthogonality’. These properties make it possible to calculate the coefficients very efficiently. There is no redundancy in the sense that there is only one possible wavelet decomposition for the signal being analyzed. However, not all basis functions have these properties. A frequently mentioned term in the definition of a basis function is ‘compact support’, which means that the values of the basis function are non-zero for finite intervals. This property enables one to efficiently represent those signals that have localized features.

### *3.2. Discrete wavelet transform*

The main idea of discrete wavelet transform (DWT) is the same as that of continuous wavelet transform (CWT). While CWT requires much calculation effort to find the coefficients at every single value of the scale parameter, DWT adopts dyadic scales and translations (i.e. scales and translations based on the powers of two) in order to reduce the amount of computation, which results in better efficiency of calculation. Filters of different cutoff frequencies are used for the analysis of the signal at different scales. The signal is passed through a series of high-pass filters to analyze high frequencies, and through a series of low-pass filters to analyze low frequencies.

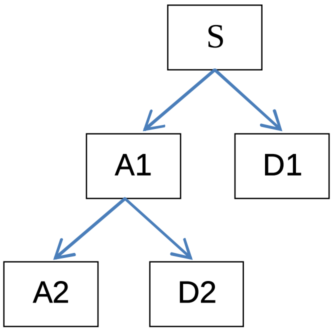



Figure 1. Wavelet tree decomposition

In DWT, the signals can be represented in an approximate or detailed form, Figure 1. The detailed form at level $j$ is defined as:

$$D_j(t) = \sum_{k=-\infty}^{\infty} cD_j(k) y_{j,k} dk \tag{10}$$

Where $cD_j(k)$ is *wavelet coefficients* at level $j$ which can be expressed as:

$$cD_j(k) = \int_{-\infty}^{\infty} f(t) j_{j,k} dt \tag{11}$$

The approximation at level $j$ is defined as:

$$A_j(t) = \sum_{k=-\infty}^{\infty} cA_j(k) j_{j,k} dk \tag{12}$$

where $cD_j(k)$ are scaling coefficients at level $j$ which is defined as:

$$cA_j(k) = \int_{-\infty}^{\infty} f(t) y_{j,k} dt \tag{13}$$

Finally, the signal $f(t)$ can be represented by:

$$f(t) = A_j(t) + \sum_{j \le J} D_j(t) \tag{14}$$

In DWT scaling and wavelet function are used. These are related to low-pass and high-pass filters, respectively.

The wavelet function can also be represented as:

$$y_{j,k}(t) = \frac{1}{\sqrt{2^j}} y\left(\frac{t - 2^j k}{2^j}\right) \tag{15}$$

The scaling function can also be represented as:

$$j_{j,k}(t) = \frac{1}{\sqrt{2^j}} j\left(\frac{t - 2^j k}{2^j}\right) \tag{16}$$

$$\left\{ J = 1,2,....., j; K = 1,2,.....k \;\middle|\; j \le \log_2^N, k = \frac{N}{2^j} \right\}$$

Not all the wavelet functions have scaling functions. Only orthogonal wavelets have their scaling functions. Since, each *Dj(t)* has a range of particular out of which the intensity is zero, a supposition is introduced here that the original function *f(t)* is decomposed into a series of *Dj(t)*'s exclusively in frequency domain. In other word, each *Dj(t)* has non-zero components only in an exclusive range of frequency. This supposition is not theoretically exact but is justified later from an engineering practicepoint of view The exclusive range of frequency of *Dj(t)* is denoted as follows:

$$\text{Frequency range of level } j = \left\lfloor f_{1j}, f_{2j} \right\rfloor \tag{17}$$

$$\text{Period range of level } j = \left\lfloor T_{1j}, T_{2j} \right\rfloor \tag{18}$$

From the nature of discrete wavelet transform, the $D_j(t)$ has components of half the frequency of $D_{j+1}(t)$. $f_{1j}$, $f_{2j}$, $T_{1j}$ and $T_{2j}$ are expressed as follows:

$$\left[ \frac{1}{2^{j+1}\Delta t}, \frac{1}{2^j \Delta t} \right] \tag{19}$$

$$\left\lfloor 2^j \Delta t, 2^{j+1} \Delta t \right\rfloor \tag{20}$$

Where $\Delta t$ is the time step of the digital data of *f(t)*.

## 4. PROPOSED METHOD

As mentioned before, the wavelet transform provides the frequency content of a signal in each time. On other hand, the ETAFs are functions where their amplitudes increase with time. In this paper, this similarity is utilizes for generation of the ETAFs.

The procedure for the generation of ETAFs in this study is provided in the following:

**Step 1**: Initialization. The initial points are considered as random values with zero mean and unit variance.

**Step 2**: Decomposition of AF. In this step the AF is decomposed with wavelet to $j = n$ levels. The number of levels depended on the frequency range. In this paper, the frequency range and the time step are 0-10s and 0.01s, respectively. Therefore, given the Eq. (20), the signal is decomposed into 9 levels using wavelet that detail coefficients covers the frequency range [0-10.24]s. It should be noted that the points are decomposed with db-10 wavelet [9]. The wavelet tree decomposition utilized in this paper is shown in Figure 2.

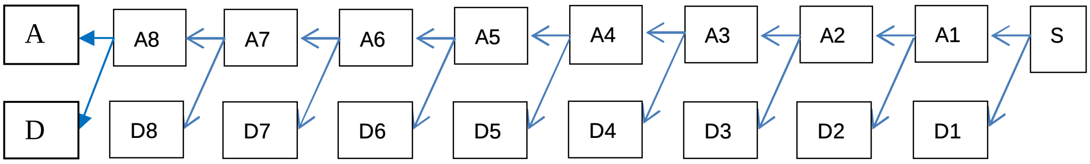



Figure 2**.** Wavelet tree decomposition utilized in this paper

**Step 3**: Division of the detail coefficients. To reduce the number of optimization variables, the detail coefficients of each level is divided to equal parts. Then, each part is multiplied by a value. In fact, these values are the variables in the optimization process. Therefore, the number of the optimization variables is equal to the number of the decomposition levels to the number of the Equal Parts (EPs). In following sections, more discussion will be provided on the number of EPs.

**Step 4**: Reconstruction. After modification of the detail coefficients in each level (in the previous step), the new acceleration function can be represented as:

$$f(t) = \sum_{j=1}^{9} D_j^{\,m}(t) \tag{21}$$

Where, $D_j$ is the detail at level j. The superscript $m$ refers to “after modification”. For controlling spectral displacement in high periods, the approximate coefficients in level 9, ($A_9$), is not considered.

**Step 5**: Determine the objective function. The objective function is the root-square-mean of the error between response and target spectrum in each time and period (Eq. (6)). In addition, damping ratio is assumed to be 5% for all of the SDOF systems.

**Step 6**: Optimization process. There are a great number of optimization techniques, such as the Quasi-Newton method and the Nelder-Mead method [14], which can be used to achieve the minimization of the objective function to make the response spectrum of the generated earthquake time-history as close to the target design spectrum as possible. For the optimization problem (6) in the proposed generation method, the non-linear Quasi-Newton algorithm is applied. The detailed description of the Quasi-Newton algorithm can be found in reference [14].

## 5. EFFECT OF THE NUMBER OF EQUAL PARTS (EPS)

The effect of the number of EPs in the error value and the function count, is presented in this section. As stated before, the detail coefficients of each level are divided to the number of EPs. Then, each variable is multiplied by a value. Undoubtedly, the error value becomes less, if high number of EPs is considered. On other hand, the number of variables and number of function count grows in the optimization process.

For this purpose, an acceleration function is generated. Duration of this AF is 20.47 seconds which consist of $2^{11}$(2048) acceleration points in 0.01s time steps when the response of a SDOF system equals to the codified template design spectrum with a scale factor of unity. The target time of this AF is 10th second. This acceleration function is decomposed to nine levels via the wavelet transform. The base errors are evaluated in the latest iteration of the optimization process for various numbers of the EPs (1, 2, 4, 8, 16, 32 and 64). Initial signal and the variables are identical in the optimization process.

In Figure 3 and Figure 4, the effect of the number of EPs on the error and function count is plotted, respectively. As can be seen, the error and the error decrease rate are reduced by increasing the number of EPs. Therefore, the function count is increased in the optimization process by the increasing the number of EPs. After the number of EPs becomes 32, the error does not change and the function count is increased quite high. Therefore, the number of EPs should not be considered higher than 32 in this procedure.

One idea to reduce the function count is to increase the number of EPs after some iterations. In order to utilize the above explanation, two variants are designed:

Method I: The number of EPs is constant in all iterations.

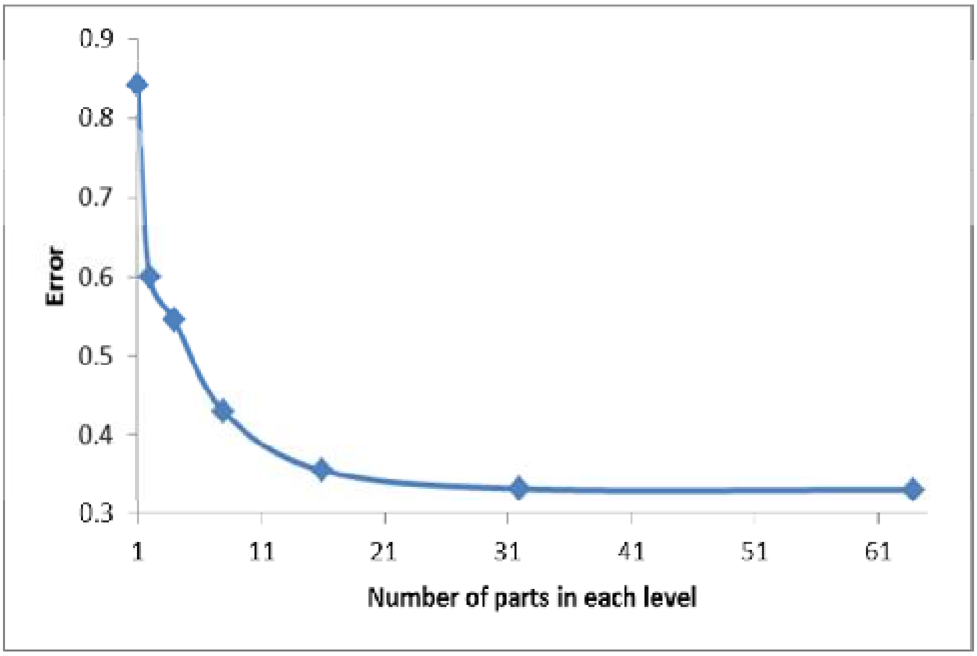



Figure 3. Effect of the number of EPs on the error

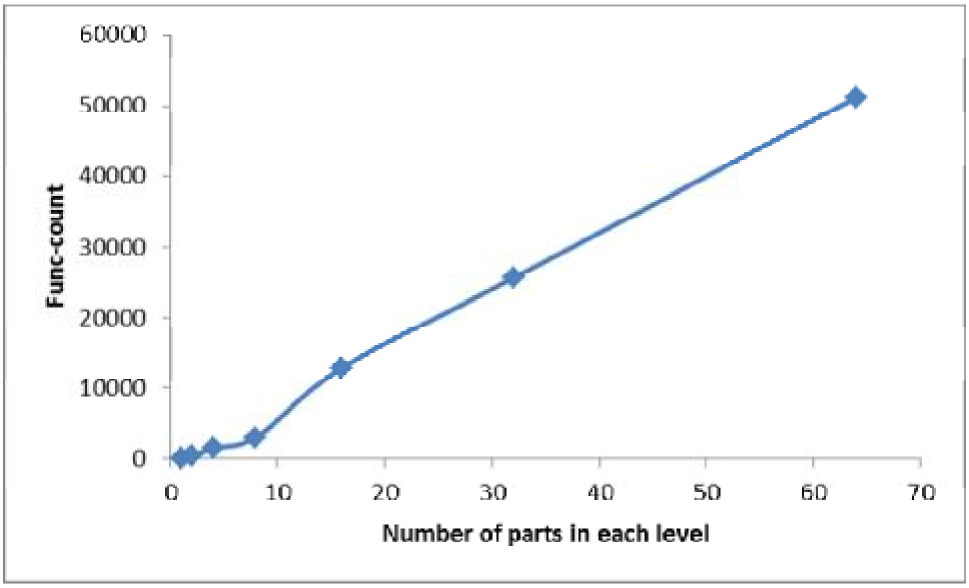



Figure 4. Effect of the number of EPs on the function count

Method II: In this method, the optimization iterations are divided to some steps. In the first step the number of EPs is taken as 1, and in subsequent steps this number is increased to twice the number of EPs in the previous step.

## 6. NUMERICAL EXAMPLES

In this study, two AFs and for each AF three series are generated. Maximum fundamental period of the system considered to be 10 seconds. The periodic range from 0.0 to 10.0 second is divided into 0.2-second time steps. A mathematical programming procedure, which applies a quasi-Newton method, is used to produce this acceleration functions. In this method, the number of function count is equal to the number of variables in each iteration. The optimization process is performed by a Core™ 2 Duo 2.53 GHz computer and the time for all the computations is evaluated in clock time.

### *6.1. Twenty second acceleration functions*

In this example, for illustrating the proposed method an AF is generated so as to be compatible with the EC8 design spectrum for soil type (B) and 0.35g peak ground. Duration of these AF is 20.47 seconds which consist of $2^{11}$ (2048) acceleration points in 0.01s time steps. The target time of this AF is 10th second when the response of a SDOF system with a damping ratio of 5% equals to the codified template design spectrum with a scale factor of unity.

For the generation of this AF we have used Methods I and II. In Tables 1 and 2, the number of variables and the number of EPs are shown. It can be seen that the utilization of the Method II reduces the function count by more than 37 percent. It should be noted that for generating this AF in time domain with 150 optimization process iterations, the number of function count is equal to 307,200 (=150*2048).

Table 1. Specifications of the Method I

| Iteration | 0-150 | sum of function count |
|---|---|---|
| number of EPs | 32 | |
| Number of variables | 288 | |
| Function count | 43,200 | 43,200 |

Table 2. Specifications of the Method II

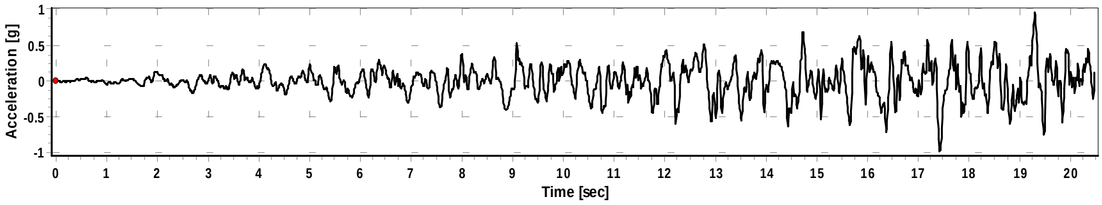
| Iteration | 0-10 | 11-20 | 21-35 | 36-50 | 51-75 | 76-150 | Sum of function counts |
|---|---|---|---|---|---|---|---|
| Number of EPs | 1 | 2 | 4 | 8 | 16 | 32 | |
| Number of variables | 9 | 18 | 36 | 72 | 144 | 288 | |
| Function count | 90 | 180 | 540 | 1080 | 3600 | 21600 | 27,090 |

Performances are assessed on the basis of the base error average values and the statistics results of the both methods as reported in Table 3. The initial points of each series are equal in both methods. The average base error and function count obtained by the Method II is less than those of Method I. It can be seen that the utilization of the three AF reduces the error by more than 38 percent as well. The CPU-time consumption of the program, coded in MATLAB®, is roughly 3 hours for generating these AFs using Method II.

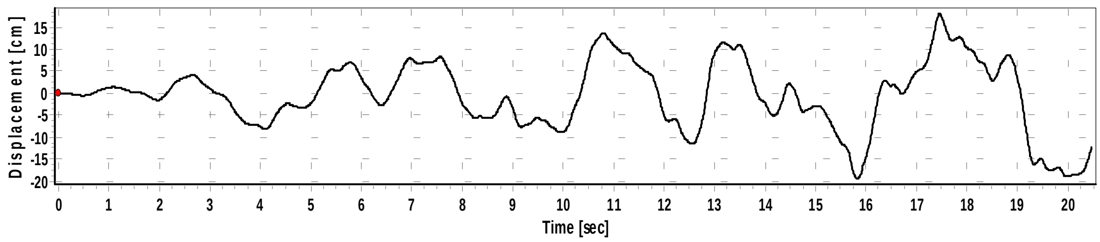
Table 3. Base error for the Twenty second AF responses

| Acceleration Function | Error | |
|---|---|---|
| | Method I | Method II |
| Series 1 | 0.3558 | 0.3306 |
| Series 2 | 0.3605 | 0.3530 |
| Series 3 | 0.3512 | 0.3400 |
| Average | 0.3558 | 0.3412 |
| Ave ETAF | 0.2912 | 0.2156 |
| Function count | 43,200 | 27,090 |

The produced best endurance time acceleration function (Series 1 of Method II) are labeled as ETA20WII-01. The ETA20WII-01 acceleration and displacement function generated utilizing this method is shown in Figure 5. The average acceleration and displacement response spectra of the ETA20WII functions are illustrated in Figure 6 and Figure 7, respectively.




(a)




(b)

Figure 5. ETA20WII-01 acceleration (a), displacement (b) function

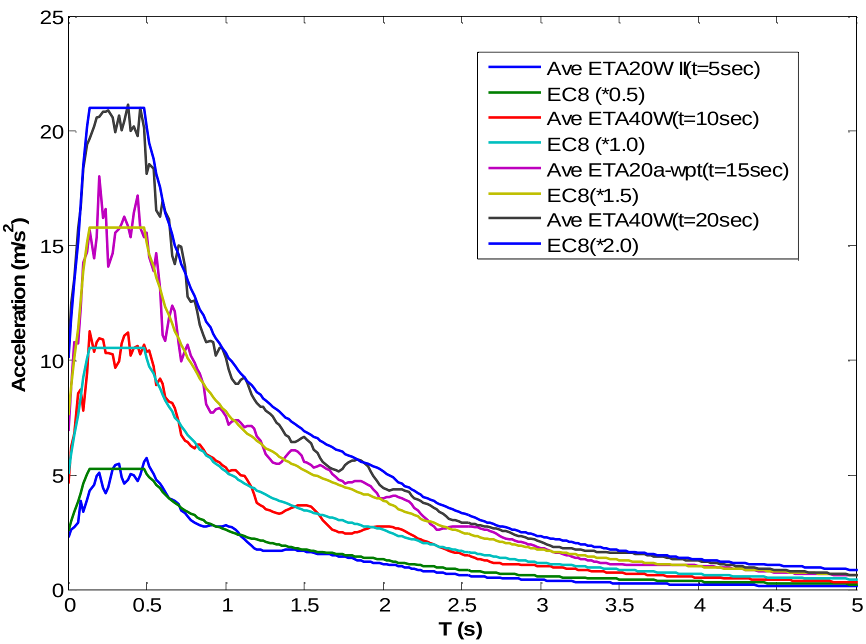



Figure 6. Acceleration response spectrum of the ETA20WII at 5th, 10th, 15th and 20th seconds

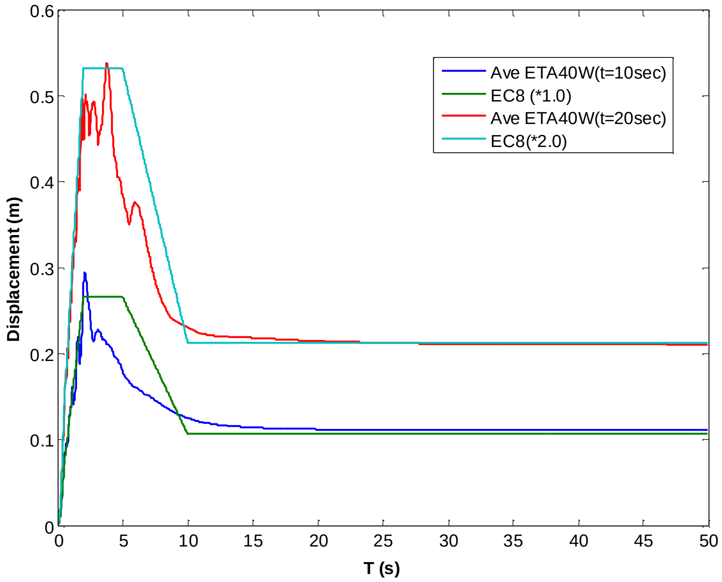



Figure 7. Displacement response spectrum of the ETA20WII at 10th and 20th seconds

As can be seen from Figure 7, due to disregarding the approximate coefficients in latest decomposition level of wavelet transform, the displacement response spectrum becomes approximately constant in periods higher than T=10 sec without considering the long period in optimization process.

In Figure 8, the convergence curves of both methods are presented for the generation of series 1 of this AF. As can be seen, in Method II the convergence curve has a stepped form. This indicates the efficiency of the Method II since in low iterations the convergence is higher, and the number of the function count is lower than those of Method I.

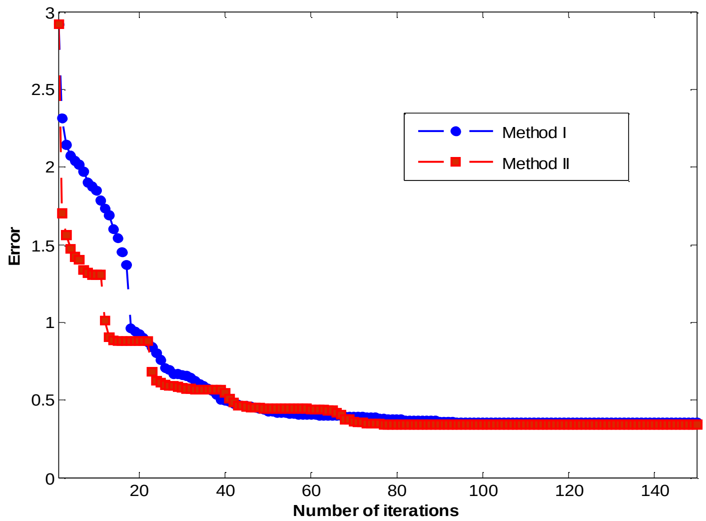



Figure 8. Comparison of the convergence of methods I and II in optimization of acceleration function

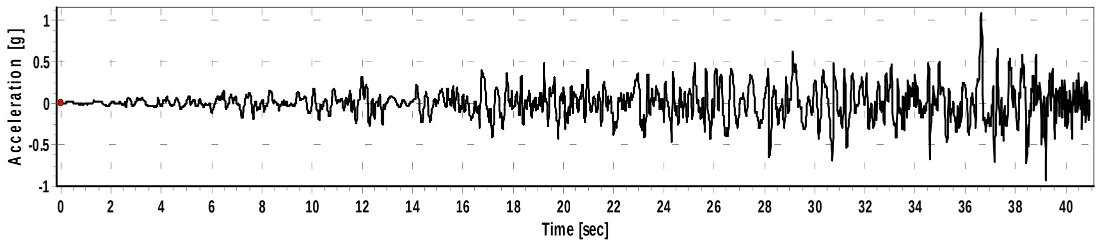
### *5.1. Forty second acceleration functions*

Duration of the ETAF produced in the previous examples was 20 seconds. If the duration of the AF is increased from 20 seconds to 40 seconds, the number of acceleration points will be increased from 2048 points to 4096 points. Thus, the number of optimization variables will be 4096 in the time domain optimization. Consequently, the efficiency of the optimization procedure seriously declines due to significant increases in the computational time. Therefore, to produce AF with longer duration in the time domain optimization, other techniques are reported in [8]. However, in this study we will not deal with this problem due to decreasing the optimization variables. The number of variables and EPs are provided in Table 4.

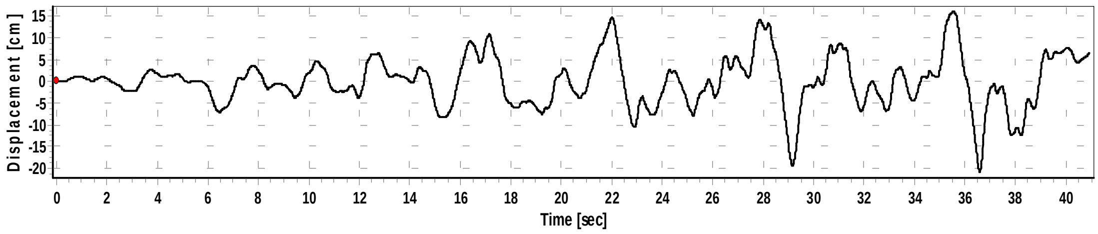
Table 4. The number of variables and function count for generating Forty second AF

| **Iteration** | **0-10** | **11-20** | **21-35** | **36-50** | **51-75** | **76-100** | **101-150** | **Sum of the function count** |
|---|---|---|---|---|---|---|---|---|
| Number of EPs | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
| Number of variables | 9 | 18 | 36 | 72 | 144 | 288 | 576 | |
| Function count | 90 | 180 | 540 | 1,080 | 3,600 | 7,200 | 28,800 | 41,490 |

From Table 5, it can be seen that the utilization of the three AF reduces the error by more than 30 percent as well. The CPU-time consumption of the program, coded in MATLAB®, is roughly 8 hours for generating this AF.




(a)




(b)

Figure 9. ETA40W-01 acceleration (a) displacement (b) function

Table 5. Base error for the AF responses of the long duration of the ETAFs

| Acceleration function | Error |
|---|---|
| Series 1 | 0.3401 |
| Series 2 | 0.3631 |
| Series 3 | 0.3719 |
| Average | 0.3583 |
| Ave ETAF | 0.2403 |
| Function count | 14,100 |

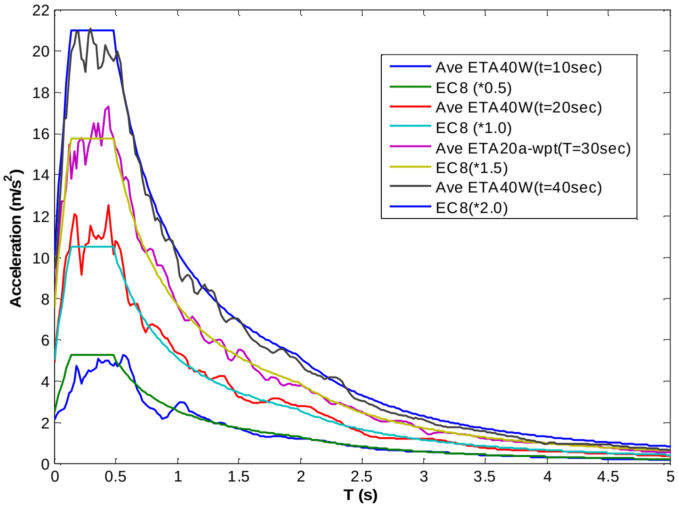



Figure 10. Acceleration response spectra of the ETA40W at 10th, 20th, 30th and 40th seconds

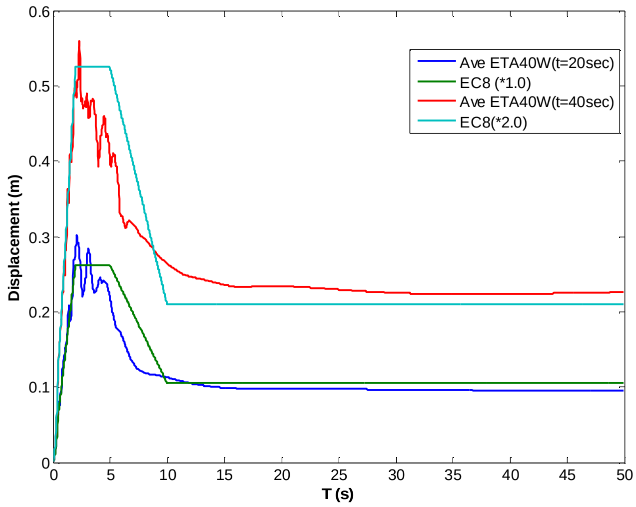



Figure 11. Displacement response spectra of the ETA40W at 20th and 40th seconds

The best produced 40s endurance time acceleration function is labeled as ETA40W-01. The ETA40W-01 acceleration function generated utilizing this method is shown in Figure 9. The average acceleration and displacement response spectra of the ETA40W functions are illustrated in Figure 10 and Figure 11, respectively.

## 7. CONCLUDING REMARKS

Wavelet transform is a good tool adaptive to time-frequency analysis in earthquake engineering. On the other hand, endurance time (ET) method is a new dynamic pushover procedure for seismic assessment and structural design, in which structures are subjected to a predesigned intensifying excitation function and their performance is assessed based on the length of the time interval that they can satisfy the required performance objectives. Generating efficient dynamic functions is essential for success of the ET method. Up to now, for generating the ET acceleration functions only the time domain modification procedure is used and this procedure is a time consuming process.

In this paper, a new method is presented for the generation of ETAFs compatible with the European Code regulations (EC8) in time and frequency domains. For modification of data in frequency domain wavelet transform is used. Also, for modification of data in time domain unconstrained optimization is utilized.

The procedure of generating the acceleration functions starts from random points. This random points are then decomposed with wavelet transform to $j = n$ levels. To reduce the number of variables, the detail coefficients of each level are divided to equal parts. Each equal part is then multiplied by a value. This value is evaluated by the optimization procedure. A mathematical programming procedure (Quasi-Newton) is utilized to produce the ETAFs. This procedure is used by two methods. In Method I, a constant number of variables is considered in the all optimization iterations, and in Method II the number of variables grows during the optimization iterations. Method II reaches to smaller error and lower number of function evolutions is needed compared to Method I.

Comparing the results with other ETAFs demonstrates that the proposed approach have good capability of determining the optimum solutions. This procedure finds the optimum results in a small number of function count. As a result, the presented new method is powerful optimization method which can easily be expanded for the generation other ETAFs compatible with other code spectrum.

## REFERENCES

1. Kaveh A, Farahmand Azar B, Hadidi A, Rezazadeh F, Talatahari S. Performance-based seismic design of steel frames using ant colony optimization, *J Construct Steel Res*, 2010; **66**(4): 566-74.
2. Bozorgnia Y, Bertero VV. *Earthquake Engineering: From Engineering Seismology to Performance-Based Engineering*, CRC Press, USA, 2004.
3. Gong Y, Xu L, Grierson DE. Performance-based design sensitivity analysis of steel

moment frames under earthquake loading, *Int J Numer Meth Eng*, 2005; **63**(9): 1229-49.
4. Moghaddam H. Earthquake engineering: theory and application, Farahang Publications, Tehran, Iran, 2002.
5. Estekanchi HE, Vafai A, Sadeghazar M. Endurance time method for seismic analysis and design of structures, *Scientia Iranica*, 2004; **11**(4): 361-70.
6. Estekanchi HE, Valamanesh V, Vafai A. Application of endurance time method in linear seismic analysis, Eng Struct, 2007; **29**(10), 2551-62.
7. Riahi HT, Estekanchi HE. Seismic assessment of steel frames with endurance time method, *J Construct Steel Res*, 2010; **66**(6):780-92.
8. Nozari A, Estekanchi HE. Optimization of endurance acceleration functions for seismic assessment of structures, *Int J Optim Civil Eng*, 2011; **2**:257-77.
9. Daubechies I. Ten lectures on wavelets. CBMS-NSF Conference Series in Applied Mathematics, Montpelier, Vermont, 1992.
10. Newland DE. *Random Vibrations, Spectral and Wavelet Analysis*, 3rd Edition, Longman, Singapore Publishers, 1994.
11. Rajasekaran S, Latha V, Lee SC. Generation of artificial earthquake motion records using wavelets and principal component analysis, *J Earthquake Eng*, 2006; **10**(5): 665-91.
12. Ghodrati Amiri G. Ashtari P, Rahami H. New development of artificial record generation by wavelet theory, *Struct Eng Mech*, 2006; **22**(2):185-95.
13. CEN. Eurocode 8: Design of Structures for Earthquake Resistance- Part 1: General Rules, Seismic Actions and Rules for Buildings. EN 1998-1: 2003 E. ComitEuropde Normalisation, Brussels, 2003.
14. Nocedal J, Wright SJ. *Numerical Optimization*. Springer, New York, 1999.