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JOINT ECONOMIC LOT SIZING PROBLEM IN A TWO ECHELON PRODUCTION SYSTEM WITH FINITE PRODUCTION RATE AND LEAD TIME

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ABSTRACT

In this study, a two-echelon supplier-manufacturer system with finite production rate and lead time is proposed. It is assumed that shortage is not permitted and the lot size of manufacturer (second echelon) is *m*-factors of the lot size of supplier (first echelon) and supplier can supply the manufacturer's lot size in several shipments in each cycle. So, the production rate of supplier is greater than manufacturer's. The proposed model aims to determine the optimal lot-size of each echelon such that the total cost of system is minimized. First, the problem is studied regardless of lead time and the optimal value of the lot sizes and the number of shipments is determined through analytical relations. Then, an exact solution algorithm for the problem is presented for the case with non-zero lead time. Finally, the performance of the proposed algorithm is reviewed by solving some numerical instances of the problem.

Keywords: Inventory control; Joint economic lot sizing; multi echelon production systems; finite production rate.

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1. INTRODUCTION

In today's competitive business environment, most of the companies outsource their processes which does not relate to their main production scope in order to concentrate on their specialized area. Moreover, these processes have low added value and expert companies can produce them with lower cost. In most industries, 60 percentages of activities

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are outsourced, approximately [1]. This shows the importance of interaction between the components of a supply chain. Interaction, planning an inventory program and joint production between the components of supply chain can enhance the customer service and leads to the component's profit in long term. Goyal [2] considered a "single vendor- single buyer" model in which the vendor's production rate is infinite. The model minimizes total cost of the system. Banerjee [3] developed the model of Goyal by assuming finite production rate and Lot-for-Lot policy for the vendor. Goyal [4] developed the model of Banerjee more comprehensive by relaxing the assumption of Lot-for-Lot policy. Monahan [5] presented a "single supplier-single retailer" model in which the supplier supplies the retailer's demand bye Lot-for-Lot policy. He showed that the supplier can increase his revenue by offering a discounted price to the retailer and encouraging him to raise his lot size. Monahan studied the values of decision variables from the retailer's and supplier's viewpoint separately, in place of the viewpoint of an integrated supply chain system. Beullens and Janssens [6] studied the model of Monahan by net present value approach. Lu [7] presented a "single vendor-multiple buyer" system in which total cost of vendor is minimized by considering budget contraints for each of the buyers. He assumed that the size of the sub shipments shiped in a cycle are equal and presented a heuristic algorithm for determining the optimal value of lot size. Goyal [8] developed the model of Lu by considering that the size of sub shipments in a cycle increases by the ratio of " vendor's production rate divided by buyer's demand rate", successively and the goal is to determine the lot size of first sub shipments. Hill [9] studied the model of Lu in a more comprehensive condition by assuming the size of consecutive shipped sub shipments increase by a constant ration. Goyal and Nebebe [10] presented an alternative solution method for the model of Hill. Hill [11] developed an algorithm by combining the equal shipments policy (Lu [7]) and consecutive increase of shipment size (Goyal [8]) in order to find the optimal solution. In all these researches, the holding cost per unit for the buyer is greater than the vendor's. Hill and Omar [12] studied the model of Hill under the assumption that holding cost of the vendor is greater. Ha and Kim [13] studied a "single buyer-single vendor" system in lean production environment in order to minimize the total cost of the supply chain system. Munson and Rosenblatt [14] presented a three echelon supply chain (supplier, manufacturer and retailer) in which the retailer supplies the demand with a constant rate. In this model, production rate of supplier and manufacturer is finite. They studied the model as an integrated supply chain system and presented an algorithm to minimize the total cost of the system. Ben-Daya and Al-Nassar [15] studied a three echelon supply chain " supplier, manufacturer and retailer" and presented an algorithm in order to minimize the total cost of supply chain. They showed that if the sub shipments are shipped quickly after production, a significant saving occurs in costs in comparison with the case which shipping the sub shipments is subjected to the completion of production process of the whole lot size. Wu and Ouyang [16] presented a "single buyer-single vendor" system by considering the shortage. They developed an algebraic procedure to minimize the total cost of system, determining economic order quantity, number of shipments in each cycle and the optimal size of the shortage. Chung [17] found that the conditions provided by Wu and Ouyang is incomplete for the optimality. He modified the developed algorithm by Wu and Ouyang. Giri and Bardhan [18] developed a two echelon supply chain "supplier-manufacturer" with finite production rate for manufacturer and considering time value of money and inflation rate. Jauhari [19] presented

a buyer-vendor supply chain system by considering discount and stochastic demand following normal distribution. Kaya et al. [20] studied a retailer-supplier supply chain system in which production rate of supplier is infinite. Also, retailer does not hold inventory and supplies the constant rate demand with back order. Glock and Kim [21] studied a "single buyer-multi vendor" supply chain system. In this single commodity system, the buyer supplies the constant rate demand without shortage and provides the required commodities from several vendors. But it is possible that the vendors be classified in categories and shipped lot size of each category receives to the buyer simultaneously. Sadjadi et al. [22] developed a vendor-buyer supply chain system by assuming finite production rate for the vendor and budget constraints. For a more comprehensive review of lot size determination in multi echelon supply chain models see Glock [23].

The remainder of the paper is as follow. The problem and its related assumptions are defined in Section 2.

2. PROBLEM DEFINITION

Consider two echelon production system consisted of a supplier, as the first echelon, and a manufacturer as the second echelon where their production rate is finite. Manufacturer receives raw material from the supplier and produces finished products with consumption rate 1 to satisfy the external demand without shortage, therefore, production rate of the supplier is greater than the manufacturer's which is greater than the external demand rate. In the proposed problem, it has been assumed that the lot size of the manufacturer is *m*-factors of the supplier's lot size. In other words, the manufacturer receives semi-finished products (e.g. raw materials) for *m*-times from the supplier in each production cycle. Transportation cost of each shipment is considered fixed and independent of the size of the shipment. The problem is to find the optimal lot size of the manufacturer and supplier in an infinite planning horizon both with and without lead time.

The symbols used in this remainder of the paper are defined as follows:

D : The annual rate of external demand

P_1 : Production rate of supplier

P_2 : Production rate of manufacturer

A_1 : Fixed setup cost of the supplier

C_{tr} : Fixed transportation cost of each shipment from the supplier to the manufacturer

m : The number of shipments in each cycle

Q_{1i} : Quantity of *i*th shipment in each cycle

Q_2 : Lot size of manufacturer in each cycle

L : Lead time of each shipment

h_1 : Annual holding cost per unit for the supplier

h_b : Annual holding cost of semi-finished product per unit in manufacturer's warehouse of raw materials

h_2 : Annual holding cost of finished product per unit for the manufacturer

T_2 : Cycle time of manufacturer

T_{1i} : Required time for producing the *i*th shipment by the supplier

T_{bi} : Required time for consuming the *i*th shipment in manufacturer's warehouse of raw materials

Fig. 1 depicts the inventory level both for manufacturer and supplier. In this figure, the number of shipments is 3. In general, the system cycle is equal to the manufacturer's cycle due to its smaller production rate assumption. In each cycle, supplier ships the lot size of the manufacturer (Q_2) in m shipments by the quantity of Q_{1i} ($i=1,...,m$) for each one. Due to this end, the equations below holds true:

$$T_{1,i} = \frac{Q_{1,i}}{P_1} \quad ,i=1,...,m \tag{1}$$

$$T_{b,i} = \frac{Q_{1,i}}{P_2} \quad ,i=1,...,m \tag{2}$$

$$T_2 = \frac{Q_2}{D} \tag{3}$$

$$Q_2 = \sum_{i=1}^{m} Q_{1,i} \tag{4}$$

For the purpose that the machines in second echelon does not cease the production operations, the shipment *i+1* should receive to the manufacturer before the shipment *i* had been consumed. In other words $T_{1,i+1} \le T_{b,i}$,$i=1,m-1$. So, the following conditions must hold true for the size of shipments.

$$\frac{Q_{1,i+1}}{P_1} \le \frac{Q_{1,i}}{P_2} \quad ,i=1,...,m-1 \tag{5}$$

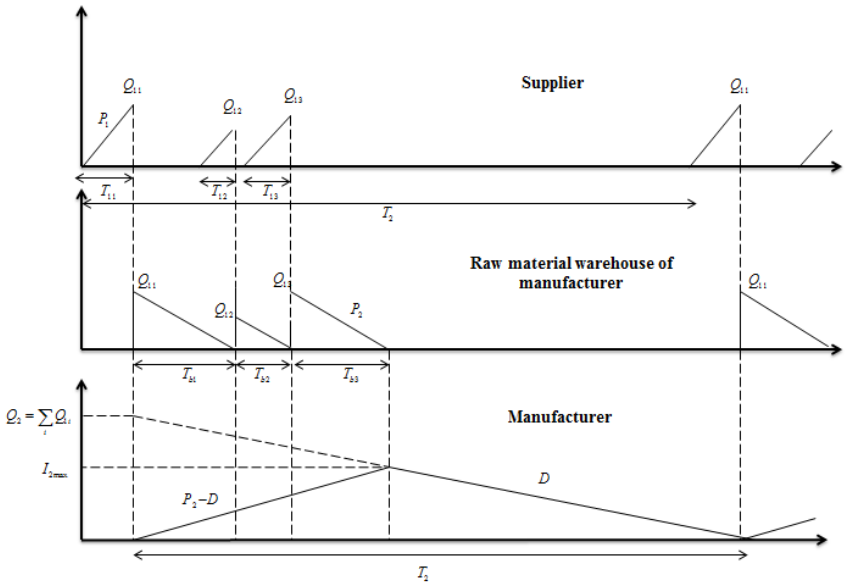

Figure 1. Inventory Level of system

3. SOLUTION APPROACH

In this section we first show an essential property of the order sizes and then we consider the problem in two cases. In the first case the lead time, the time between ordering and receiving the ordered items, is assumed to be zero and in second case the positive lead time in problem is investigated.

Before we state the property of the optimal order sizes, we need to calculate the average inventories in different echelons. Average inventory of supplier, raw material warehouse of manufacturer and the manufacturer's machine is as follows, respectively:

$$\bar{I}_1(Q_{11},...,Q_{1m}) = \sum_{i=1}^{m} \frac{Q_{1i}}{2}\frac{T_{1i}}{T_2} = \frac{D}{2P_1Q_2}\sum_{i=1}^{m} Q_{1i}^2 \tag{6}$$

$$\bar{I}_b(Q_{11},...,Q_{1m}) = \sum_{i=1}^{m} \frac{Q_{1i}}{2}\frac{T_{bi}}{T_2} = \frac{D}{2P_1Q_2}\sum_{i=1}^{m} Q_{1i}^2 \tag{7}$$

$$\bar{I}_2(Q_2) = \frac{Q_2}{2}\left(1-\frac{D}{P_2}\right) \tag{8}$$

Theorem 1. In optimal solution, the size of shipments is equal.

Proof. We prove that shipments with equal size have the minimum average cost for a certain lot size Q_2 and a specific number of shipments *m*. If Q_2 and *m* are specified, average setup cost of the supplier and manufacturer and average in-process inventory holding cost in manufacturer machines will be constant and they need not to be considered. The sum of holding cost in supplier and manufacturer sections is as follows.

$$C_{1,b}^h = h_1\frac{D}{2P_1Q_2}\sum_{i=1}^{m} Q_{1i}^2 + h_b\frac{D}{2P_1Q_2}\sum_{i=1}^{m} Q_{1i}^2 = K\sum_{i=1}^{m} Q_{1i}^2 \tag{9}$$

In Equation (9), *K* is constant which does not depend on the size of shipments. Here, we study the associated cost of the condition that all the shipment sizes are equal. In this case, the size of each shipment is $Q_1 = Q_2/m = \sum Q_{1i}/m$.

$$C_{1,b}'^h = K\sum_{i=1}^{m}\left(\sum_{i=1}^{m}\frac{Q_{1,i}}{m}\right)^2 = Km\left(\sum_{i=1}^{m}\frac{Q_{1,i}}{m}\right)^2 = K\frac{\left(\sum_{i=1}^{m} Q_{1,i}\right)^2}{m} \tag{10}$$

$$\Rightarrow C_{1,b}'^h = K\frac{\sum_{i=1}^{m} Q_{1,i}^2 + 2\sum_{i<j}^{m} Q_{1,i}Q_{1,j}}{m} \tag{11}$$

We show that $C'^{h}_{1,b} \le C^{h}_{1,b}$. Since we have $\sum_{i<j}^{m}\left(Q_{1,i} - Q_{1,j}\right)^2 \ge 0$, then

$$(m-1)\sum_{i=1}^{m} Q_{1,i}^2 - 2\sum_{i<j}^{m} Q_{1,i}Q_{1,j} \ge 0 \tag{12}$$

Therefore, we have

$$\sum_{i=1}^{m} Q_{1i}^2 \ge \frac{\sum_{i=1}^{m} Q_{1,i}^2 + 2\sum_{i<j}^{m} Q_{1,i}Q_{1,j}}{m} \Rightarrow C^{h}_{1,b} \ge C'^{h}_{1,b} \tag{13}$$

According to (13), as far as in the best case, the size of shipments is equal for each Q_2 and *m*, therefore, they will be either equal in optimal solution for Q_2^* and m^*, so:

$$Q^*_{1,i} = Q^*_1 = \frac{Q^*_2}{m^*} \tag{14}$$

3.1 The problem with zero lead time

In Fig. 2, inventory level both for manufacturer and supplier is presented. In this figure, the number of shipments is 2. Average amount of inventory of supplier per time unit, raw materials' warehouse of manufacturer and in-process inventory of manufacturer's machine are determined as follows, respectively:

$$\bar{I}_1 = \frac{mQ_1}{2}\frac{T_1}{T_2} = \frac{Q_2}{2m}\frac{D}{P_1} \tag{15}$$

$$\bar{I}_b = \frac{mQ_1}{2}\frac{T_b}{T_2} = \frac{Q_2}{2m}\frac{D}{P_2} \tag{16}$$

$$\bar{I}_2 = \frac{Q_2}{2}\left(1 - \frac{D}{P_2}\right) \tag{17}$$

Average annual setup and transportation cost of the system are determined as follows:

$$\bar{A} = \frac{m(A_1 + C_{tr}) + A_2}{T_2} = \frac{(m(A_1 + C_{tr}) + A_2)D}{Q_2} \tag{18}$$

By using equations (15) to (18), average annual cost of the system would be:

$$TC(Q_2,m)=\frac{(m(A_1+C_{tr})+A_2)D}{Q_2}+\frac{Q_2}{2m}\left(h_1\frac{D}{P_1}+h_b\frac{D}{P_2}\right)+h_2\frac{Q_2}{2}\left(1-\frac{D}{P_2}\right) \quad (19)$$

$m\in N$

Due to the convexity of $TC(Q_2,m)$ with respect to Q_2, economic order quantity of manufacturer for a specified m is obtained via derivative of $TC(Q_2,m)$ with respect to Q_2.

$$Q_2^*(m)=\sqrt{\frac{2D(m(A_1+C_{tr})+A_2)}{\frac{1}{m}\left(h_1\frac{D}{P_1}+h_b\frac{D}{P_2}\right)+h_2\left(1-\frac{D}{P_2}\right)}} \quad (20)$$

By equations (19) to (20), we have:

$$TC^*(m)=\sqrt{2D(m(A_1+C_{tr})+A_2)\left(\frac{D}{m}\left(\frac{h_1}{P_1}+\frac{h_2}{P_2}\right)+h_2\left(1-\frac{D}{P_2}\right)\right)} \quad (21)$$

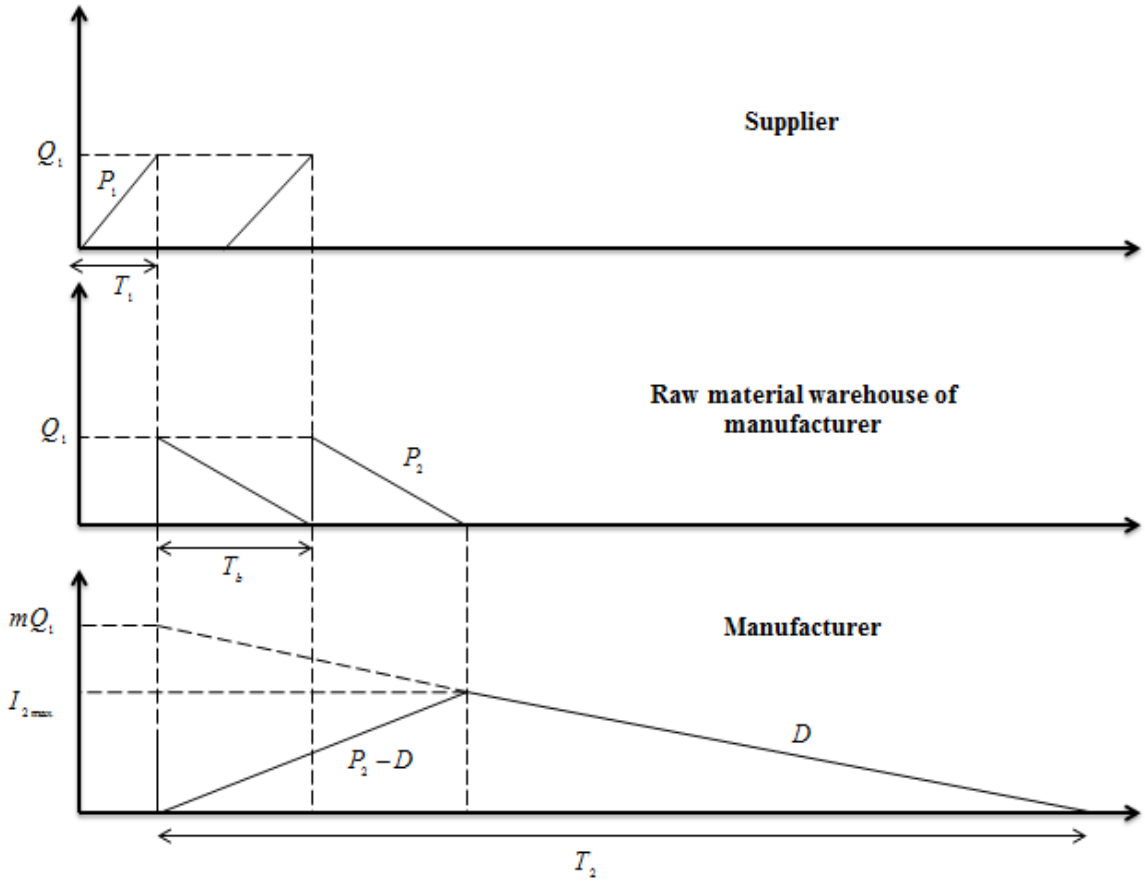

Figure 2. Inventory level of system with zero lead time

$$\frac{dTC^*(m)}{dm}=0\Rightarrow m^*=\sqrt{\frac{A_2\left(h_1\frac{D}{P_1}+h_2\frac{D}{P_2}\right)}{(A_1+C_{tr})h_2\left(1-\frac{D}{P_2}\right)}} \tag{22}$$

$TC^*(m)$ is not convex with respect to m, but its first derivative is descending for $m<m^*$ and ascending for $m>m^*$. Thus, $-TC^*(m)$ is unimodal and has global minimum in $m=m^*$. If the obtained m^* from Equation (20) is not an integer, $m'\in N$ is optimal where m' is the round up or round down of m^*. We have Theorem 2 in this regard in which the decimal part of m^* is shown by β $\left(\beta=m^*-\lfloor m^*\rfloor\right)$

Theorem 2. If m^* is not an integer, the optimal m' is obtained as follows:

If $\beta\geq 0.5$ then $m'=\lfloor m^*\rfloor+1$

If $\beta<0.5$ and $m^*<\frac{\beta(1-\beta)}{1-2\beta}$ then $m'=\lfloor m^*\rfloor+1$

If $\beta<0.5$ and $m^*>\frac{\beta(1-\beta)}{1-2\beta}$ then $m'=\lfloor m^*\rfloor$

If $\beta<0.5$ and $m^*=\frac{\beta(1-\beta)}{1-2\beta}$ then $m'\in\{\lfloor m^*\rfloor,\lfloor m^*\rfloor+1\}$

Proof. Equation (21) can be written as follows.

$$TC^*(m)=\sqrt{2D}\sqrt{F+mU+\frac{V}{m}} \tag{23}$$

In Equation (23) we have

$$F=(A_1+C_{tr})D\left(\frac{h_1}{P_1}+\frac{h_b}{P_2}\right)+A_2h_2\left(1-\frac{D}{P_2}\right) \tag{24}$$

$$U=(A_1+C_{tr})h_2\left(1-\frac{D}{P_2}\right) \tag{25}$$

$$V=A_2D\left(\frac{h_1}{P_1}+\frac{h_b}{P_2}\right) \tag{26}$$

Also

$$m^*=\sqrt{\frac{V}{U}} \tag{27}$$

However, $m^*+1-\beta$ and $m^*-\beta$ are a round up and a round down for m^*, respectively. Currently, the condition in which the round down of m^* is optimal, is discussed. Since

$$\sqrt{2D}\sqrt{F+(m^*+1-\beta)U+\frac{V}{m^*+1-\beta}} \geq \sqrt{2D}\sqrt{F+(m^*-\beta)U+\frac{V}{m^*-\beta}} \tag{28}$$

we have

$$m^*(1-2\beta) \geq \beta(1-\beta) \tag{29}$$

If $\beta \geq 0.5$, Inequality (29) never holds true. As a result, the round up of m^* is optimal. Also If $\beta < 0.5$, Inequality (29) will be:

$$m^* \geq \frac{\beta(1-\beta)}{(1-2\beta)} \tag{30}$$

If Inequality (30) holds true, the round down of m^* is optimal. Otherwise ($\beta < 0.5$), the round up will be optimal. Moreover, if $m^* = \beta(1-\beta)/(1-2\beta)$, both the round up and round down are optimal.

After determining m' by Theorem 2, the optimal lot size of the manufacturer is obtained by Equation (31) and then, the supplier's lot size (size of each shipment) will be $Q_1^* = \frac{Q_2^*}{m'}$.

$$Q_2^* = \sqrt{\frac{2D(m'(A_1+C_{tr})+A_2)}{\frac{1}{m'}\left(h_1\frac{D}{P_1}+h_b\frac{D}{P_2}\right)+h_2\left(1-\frac{D}{P_2}\right)}} \tag{31}$$

3.2 The problem with non-zero lead time

In this section, it is assumed that the produced shipment of semi-finished product in the first echelon is received by the raw materials' warehouse of second echelon with a delay of *L* units of time. In Fig. 3, the related inventory level for each part of system is presented. in this figure number of shipments is 3.

The average of inventory in first and second echelon's machines and raw materials' warehouse of second echelon are as the case without considering lead time. The average of inventory during the lead time is:

$$\bar{I}_L = mQ_1\frac{L}{T_2} = LD \tag{32}$$

The average cost of the system is obtained by Equation (33):

$$TC(Q_2,m)=\frac{(m(A_1+C_{tr})+A_2)D}{Q_2}+\frac{Q_2}{2m}\left(h_1\frac{D}{P_1}+h_b\frac{D}{P_2}\right)+h_2\frac{Q_2}{2}\left(1-\frac{D}{P_2}\right)+h_1LD \tag{33}$$

$m\in N$

By Equation (33), it is clear that lead time does not affect the optimal values of decision variable. According to Fig. 3, it is obvious that Inequality (34) must be held to make Q_2 and m feasible.

$$T_2\geq(m-1)T_b+T_1+L\Rightarrow\frac{Q_2^*(m)}{D}\geq(m-1)\frac{Q_1^*(m)}{P_2}+\frac{Q_1^*(m)}{P_1}+L \tag{34}$$

Therefore

$$Q_2^*(m)\geq\frac{L}{\frac{1}{D}-\frac{1}{P_2}+\frac{1}{m}\left(\frac{1}{P_2}-\frac{1}{P_1}\right)} \tag{35}$$

If Inequality (35) holds true for the obtained m' from Theorem 2 and resulted Q_2^* from Equation (31), the optimal solution is reached. Also, the minimum average total cost is:

$$TC^*(m')=h_1LD+\sqrt{2D(m'(A_1+C_{tr})+A_2)\left(\frac{D}{m'}\left(\frac{h_1}{P_1}+\frac{h_b}{P_2}\right)+h_2\left(1-\frac{D}{P_2}\right)\right)} \tag{36}$$

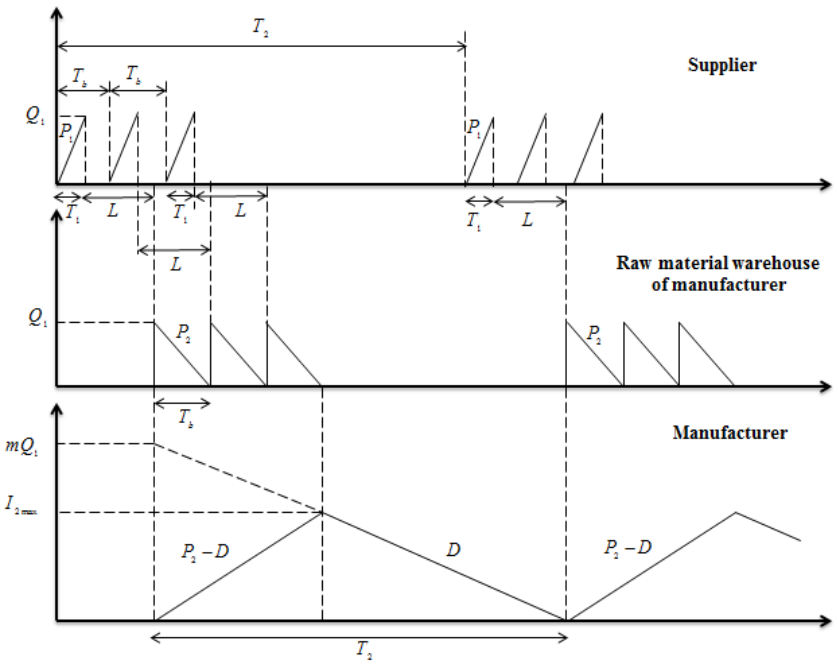

Figure 3. Inventory level of system considering lead time

Here, the condition which Inequality (35) does not hold true is studied.

$$Q_L(m)=\frac{L}{\frac{1}{D}-\frac{1}{P_2}+\frac{1}{m}\left(\frac{1}{P_2}-\frac{1}{P_1}\right)} \tag{37}$$

If the number of shipments is *m*, the obtained $Q_L(m)$ from Equation (37) is a lower bound for Q_2. Therefore, the feasible policy $m=m'$ and $Q_2=Q_L(m')$ has the minimum total cost due to the convexity of $TC(Q_2,m)$ with respect to Q_2. In Equation (37) by increasing *m*, $Q_L(m)$ increases. By choosing $m=1$ and $m=\infty$, lower and upper bound of $Q_L(m)$ can be achieved, respectively as follows.

$$Q_L^{\min}(m)=\frac{L}{\frac{1}{D}-\frac{1}{P_1}} \tag{38}$$

$$Q_L^{\max}(m)=\frac{L}{\frac{1}{D}-\frac{1}{P_2}} \tag{39}$$

According to Equation (20), by increasing *m* the value of $Q_2^*(m)$ increases infinitely. Since $Q_2^*(m)$ is bounded, if Inequality (35) does not hold true for $m=m'$, there is a m'' which Inequality (35) holds true for $m\geq m''$ and does not hold true for $m<m''$. Clearly, m'' is greater than m'.

$$Q_2^*<\frac{L}{\frac{1}{D}-\frac{1}{P_2}+\frac{1}{m'}\left(\frac{1}{P_2}-\frac{1}{P_1}\right)}\Rightarrow \exists m_R''\in(m',\infty)\,|\,Q_2^*(m_R'')=Q_L(m_R'') \tag{40}$$

$$m''=\lceil m_R''\rceil \tag{41}$$

$$\begin{cases} Q_2^*(m''+n)>Q_L(m''+n) & ,n\in N \\ Q_2^*(m''-n)<Q_L(m''-n) & ,n=1,...,m''-1 \end{cases} \tag{42}$$

In Equation (41), m'' is round up of m_R''. If $TC^*(m'')<TC(Q_L(m'),m')$, the policy of $m=m''$ and $Q_2=Q_2^*(m'')$ is optimal. Otherwise, $m=m'$ and $Q_2=Q_L(m')$ is the optimal policy.

Efforts to find m'' as a parametric form leads to a complex cubic inequality. But, upper and lower bound of m'' simply can be determined.

$$\sqrt{\frac{2D\left(m\left(A_1+C_{tr}\right)+A_2\right)}{\frac{1}{m}\left(h_1\frac{D}{P_1}+h_2\frac{D}{P_2}\right)+h_2\left(1-\frac{D}{P_2}\right)}}\geq Q' \tag{43}$$

Therefore

$$m\geq\frac{Q'h\left(1-\frac{D}{P_2}\right)-2DA_2+\sqrt{\left(Q'h_2\left(1-\frac{D}{P_2}\right)-2DA_2\right)^2+8Q'D^2\left(A_1+C_{tr}\right)\left(\frac{h_1}{P_1}+\frac{h_b}{P_2}\right)}}{4D\left(A_1+C_{tr}\right)} \tag{44}$$

$$M\left(Q'\right)=\left\lceil\frac{Q'h\left(1-\frac{D}{P_2}\right)-2DA_2+\sqrt{\left(Q'h_2\left(1-\frac{D}{P_2}\right)-2DA_2\right)^2+8Q'D^2\left(A_1+C_{tr}\right)\left(\frac{h_1}{P_1}+\frac{h_b}{P_2}\right)}}{4D\left(A_1+C_{tr}\right)}\right\rceil \tag{45}$$

$$m''_{\max}=M\left(Q_L^{\max}\right) \tag{46}$$

$$m''_{\min}=M\left(Q_L(m')\right) \tag{47}$$

Equation (46) has been written based on the fact that upper bound of $Q_L(m)$ is equal to $Q_L^{\max}$. In addition, Equation (47) has been written due to the fact that m' is a lower of m'' and does not holds true in Inequality (35). According to lower and upper bound of m'', an exact algorithm is presented for determining the optimal policy which is described as follows:

Algorithm 1.

Step 1: Calculate m^* by Equation (22) and determine m' according to Theorem 2. Then, calculate Q_2^* by Equation (31).

Step 2*:* If Inequality (35) holds true for m' and Q_2^*, the optimal solution has been reached and stop the algorithm. Otherwise, go to step 3.

Step 3: Calculate $m''_{\min}$ and $m''_{\max}$ from the equations (46) and (47) and set $k=m''_{\min}$.

Step 4: Calculate $Q_L(k)$ and $Q_2^*(k)$ by equations (37) and (20), respectively. If Inequality (35) holds true for *m=k* and $Q_2^*(k)$ then, go to step 6. Otherwise, set *k=k+1* and go to step 5.

Step 5: If $k<m''_{\max}$, go to step 4; otherwise go to step 6.

Step 6: Calculate $TC(Q_L(m'),m')$ and $TC^*(k)$ by equations (33) and (36), respectively. If $TC^*(k)<TC(Q_L(m'),m')$, the policy of *m=k* and $Q_2=Q_2^*(k)$ is optimal; Otherwise, $m=m'$ and $Q_2=Q_L(m')$ is the optimal policy and stop the algorithm.

Note that for $k = m''_{\max}$ Inequality (35) holds true. Thus, if $k = m''_{\max}$ in step 5, we go to step 6 directly without testing Inequality (35).

4. NUMERICAL RESULTS

In this section, 4 instances with zero lead time and 4 instances with non zero lead time are generated and solved. Table 1 and Table 2 show the parameters and experimental results relating to each category of instances, respectively. To show how the presented algorithm performs, we explained the way that example 6 has been solved in detailed. By Equation (22) m^* is obtained 2.5981, and then according to Theorem 2, m' is obtained as 3. Inequality (35) does not hold true for $Q_2^*(3)$ and $Q_L(3)$ obtained by equations (31) and (37), respectively. ($Q_2^*(3) = 328.6335$, $Q_{0.08}(3) = 360$). Therefore, in the next step $m''_{\min}$ and $m''_{\max}$ are calculated by equations (46) and (47) as 4 and 6 , respectively. Then, for *k=4*, $Q_2^*(k)$ and $Q_L(k)$ calculated by equations (31) and (37), respectively ($Q_2^*(4) = 394.0737$, $Q_{0.08}(4) = 384$). Inequality (35) holds true. At last, $TC(Q_L(m'), m')$ and $TC^*(k)$ are obtained by equations (33) and (36), respectively:

$TC(m = 3, Q_2 = 360) = 3550$

$TC^*(m = 4) = 3594.1$

It is clear that the policy of *m=3*, $Q_2 = 360$ and $Q_1 = 120$ is optimal.

Table 1: Parameters and experimental results of 4 examples with zero lead time

Pr.	A_1	C_{tr}	A_2	h_1	h_b	h_2	P_1	P_1	D	m^*	m'	Q_2^*	Q_1^*	TC^*
1	90	10	150	10	10	20	1500	1200	1000	2.598	3	328.634	109.545	2738.6
2	90	10	120	10	10	20	1500	1200	1000	2.323	2	243.057	121.529	2633.1
3	90	10	135	10	10	20	1500	1200	1000	2.464	3	323.110	107.703	2692.6
4	90	10	140/3	10	10	20	1500	1200	1000	$\sqrt{3}$	2	248.069	124.035	2687.4
											3	322.490	107.497	2687.4

Table 2: Parameters and experimental results of 4 examples with non-zoro lead time

Pr.	A_1	C_{tr}	A_2	h_1	h_b	h_2	P_1	P_2	D	L	Q_2^*	Q_1^*	TC^*
5	90	10	150	10	10	20	1500	1200	1000	0.06	328.634	109.545	3338.6
6	90	10	150	10	10	20	1500	1200	1000	0.08	360	120	3550
7	90	10	150	10	10	20	1500	1200	1000	0.1	450	150	3875
8	120	20	150	18	18	20	1210	1200	1000	0.06	362.565	91.641	4996.9

5. CONCLUSION

In this paper, calculating of Joint economic Lot-Sizes for a two-echelon supplier-manufacturer system was studied. In this problem, the supplier can ship the lot size of the manufacturer in several shipments. It was proved that the size of the shipments in optimal

solution is equal. First, the problem was studied with zero lead time and the optimal value of lot sizes and the number of shipments were found through analytical relations. Then, an exact algorithm was presented for the non-zero lead time case. In addition, some numerical instances were generated in order to show how the proposed algorithm performs. For the future studies, considering transportation cost depended to the shipment size (e.g. stair function) and considering multi suppliers or multi manufacturers are suggested.

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