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RELATIONSHIP OF TENSILE STRENGTH OF STEEL FIBER REINFORCED CONCRETE BASED ON GENETIC PROGRAMMING

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ABSTRACT

Estimating mechanical properties of concrete before designing reinforced concrete structures is among the design requirements. Steel fibers have a considerable effect on the mechanical properties of reinforced concrete, particularly its tensile strength. So far, numerous studies have been done to estimate the relationship between tensile strength of steel fiber reinforced concrete (SFRC) and other SFRC characteristics using regression analyses. But, in order to determine appropriate relations according to these methods, we need to estimate the basic structure of relations. Genetic programming (GP) method has solved this problem. In this study, the results of 367 laboratory specimens collected from the literature are used to present some relations to predict the tensile strength of SFRC using GP. The proposed relations are more accurate than the relations which have been presented thus far.

Keywords: reinforced concrete; steel fibers; genetic programming; tensile strength.

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1. INTRODUCTION

Using small separated fibers which is known as reinforcement of fragile materials has been identified for 1000 of years and dates back to the ancient Egypt. But, the industrial era of using fiber concrete was started by publishing the results of Romualdi and Batson's [1] research in 1963. This study proved the feasibility of using fiber to improve the plasticity and tensile strength of concrete. Thus far, various fibers with natural and artificial origins

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have been used. Due to high modulus of elasticity and tensile strength as well as resistance to alkali attacks (in the case of controlling crack width), steel fibers are more appropriate than other fibers [2]. Shah and Rangan [3] observed that use of steel fibers in concrete increased flexural stiffness, tensile stiffness, and concrete performance against tensile stresses.

Various parameters affect the behavior of steel fiber reinforced concrete (SFRC). Fiber length (L_f) , its aspect ratio $(\frac{L_f}{D_f})$, its location and tensile strength, volume ratio of fiber to concrete (V_f), and finally compressive strength of concrete have considerable impact on SFRC performance. Fiber length should be limited in specific applications, because it might cause performance problems in the concrete. ACI 318-11 building Code [4] has introduced minimum steel fibers in the beams made of SFRC equal to 0.75% of fiber to concrete volume ratio. If the volume ratio of steel fibers is selected to be more than 2.5% in concrete, there is the probability of forming fiber balls when concrete is mixed [5].

Volume and size aspect ratios of steel fiber are among the important factors for improving SFRC performance [6-7]. Generally, steel fibers have high tensile strength; in this condition, there is no rupturing probability of brittle fibers and rupturing is projected as a result of pull-out [8]. So, by increasing the strength of concrete matrix and fiber's size aspect ratio, the adhesion amount of matrix to fiber is increased, which itself increases tension in fiber. Pull-out process increases the plasticity and energy absorption of concrete. In designing concrete structures, preliminary estimation of compressive and tensile strength is to use splitting tensile test (Brazilian method). But, before designing, it is necessary to provide reliable estimation for this parameter. Identifying relations for concrete characteristics not only is useful in the design phase, but also makes it possible to achieve an optimum mixture with desirable specifications.

2. SPLITTING TENSILE STRENGTH OF SFRC

So far, several studies have been done to develop a relation between splitting tensile strength and other basic parameters of SFRC. Some of the existing relations are based on limited specimens or a specific range of fiber and aspect ratio, which constrains their application. Below, some of the studies that have led to more comprehensive relations are presented.

Thomas and Ramaswamy [9] reviewed the results of 60 specimens and introduced Eq. (1) to predict tensile strength of SFRC.

$$f_{sp} = 0.63 \sqrt{\dot{f}_{cu}} + 0.288 \times \sqrt{\dot{f}_{cu}} \times RI_V \tag{1}$$

In the above relation, f_{sp} , f_{cu} , and RI_V are splitting tensile strength of SFRC, cubic compressive concrete strength, and fiber index, respectively. Fiber index is determined according to the size and volume ratio of steel fiber using Eq. (2).

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$$RI_V = V_f \times \frac{L_f}{D_f} \tag{2}$$

Rajoub and Mohammed [10] reviewed the results of 150 laboratory samples and introduced Eq. (3) as a comprehensive relation for SFRC with different levels of resistance and aggregates.

$$f_{sp} = (0.63 + 0.46 \times RI_V) \times \sqrt{f_c} \tag{3}$$

Eq. (4) is presented based on Nadiya and Saffar's [11] study on the results of 52 laboratory and collected specimens with normal compressive strength class.

$$f_{sp} = 0.104 \times f_c + 0.00795 \times RI_V \tag{4}$$

Musmer [12] proposed the following relation to predict the tensile strength of SFRC based on the results of 358 laboratory samples collected from the literature using regression analysis.

$$f_{sp} = (0.614 + 0.4 \times RI_V^{1.029}) \times \sqrt{f_c}$$
(5)

It is noteworthy that, according to the author's studies, all the equations presented so far have been developed based on regression analysis using experimental results. In this equation developing method, it is generally necessary to determine a primary structure of relation before its development. Selecting the basic structure of relations is somehow limiting them to achieve the best response. In this paper, genetic programming method is used to determine the tensile strength relation of SFRC. For this purpose, the laboratory results of 20 sources based on 367 tests are collected (Appendix A). Variety and scope of laboratory values make it possible to achieve a comprehensive relation using GP method.

3. GENETIC PROGRAMMING (GP)

Developing educable and reliable artificial intelligence is very important for modeling practical issues when classical mathematics or statistical methods are unable to provide accurate models for the phenomena. Genetic programming is one of the youngest models in the area of computational intelligence research known as evolutionary computation [13]. GP is an evolutionary computational method for automatically solving the problems without any need for the user to know or specify the response form or structure. In contrast to intelligent computational methods such as neural network, this method will not result in a black box and its response is a mathematical relation [14].

GP was first introduced by Koza [13]. This method is used to generate regular and conceptual relations and has been used in many applications such as exponential and classic regression [15-16]. The key to this method is the use of tree structure for expressing a mathematical relation. Figs. 1 and 2 show the tree view of some hypothetical mathematical

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relations. Every relation in GP is a person who is introduced with his/her unique genetic sequence. In GP, a society with different people is considered and GP operators are used to produce the next generations. Several operators have been introduced for this method, two standard forms of which are mutation and crossover (sexual reproduction).

3.1 Mutation operator

To produce the next generations in this operator, a person is selected as a parent. A subbranch of the parent relation is randomly removed. Then, another sub-branch is randomly generated and replaced (Fig. 1).



Figure 1. Applying mutation operator in GP [17]

3.2 Crossover operator

This operator is used to combine the genetic sequence of two individuals as a parent. In this combination, a new generation of parents is generated by exchanging two random subbranches of parents (Fig. 2).

Koza [13] explained GP function in 4 steps (Fig. 3).



Figure 2. Applying Crossover operator in GP [17]



Figure 3. Relation developing steps using GP [13]

GP aims to find a very appropriate relation in the response space. Producing an initial population is a blind and random search for finding responses which is directed by GP process. The size of output relation tree resulted from GP should be limited; otherwise, considering the problem-solving process, very long and unusable relations can be expected.

4. RELATION DEVELOPMENT OF SPLITTING TENSILE STRENGTH OF SFRC

In this study, GP was used to extract relations for predicting the tensile strength of SFRC. To explain the GP model, it is necessary to feed some input and output data to the model to derive the mathematical relation. For this purpose, the results of 367 laboratory specimens were collected and used from 20 references (Appendix A). Considering that conventional concrete has some tensile strength, to consider its effect, the specimens used to determine the tensile strength of SFRC included concrete with and without fiber. All the compressive strength mentioned in Appendix A was related to the standard cylindrical sample or was modified to the standard value using Table 1. Data were randomly divided into two groups including 287 and 80 samples. These groups were respectively used for relation development (training data) and investigating the accuracy of the proposed relation obtained from GP (experimental data). To investigate the effect of various parameters on the tensile strength of SFRC, 3 groups of data were analyzed using different input parameters (Table 2).

Tuble 1. Transform coefficients to 150,500 min cylination samples [20]										
•	Cylinders 75 × 150 mm		Cylinders $100 \times 200 \ mm$	0.97	Cylinders $150 \times 300 \ mm$	1				
Cube	S	0.78	Cubes	0.0	Cubes	0.02				
$100 \times 100 \times$	100 mm	0.78	$150 \times 150 \times 150 mm$	0.8	$200 \times 200 \times 200 mm$	0.83				
	Table 2: Different groups for developing tensile strength relation									
Group name	roup name Input parameters									
Group A	Group A RI_V and cylindrical compressive strength									
Group B	RI_{V} , cylinder compressive strength, and volume ratio of steel fiber									
Group C	Volume and size aspect ratio of fiber and cylindrical compressive strength									

Table 1: Transform coefficients to 150×300 mm cylindrical samples [20]

To remove the size effect, the laboratory data were divided by their maximum amount to have a value between 0 and 1. This point should be observed in the obtained relations.

4.1 Settings of GP code

To model GP, the code sets prepared by Silva [18] were used after making some modifications. To reduce the effect of specimens in whose testing there was significant error, absolute error value was used in GP code. In other words, the sum of absolute value of the difference between laboratory results and those obtained from relation for all the samples was calculated and regarded as the amount of each relation in GP arithmetic operations. Any relation with more absolute error would have more inappropriate results. The probability of selecting parents for the use of operators was considered by paying attention to their accuracy ranking in terms of problem estimation [19]. Mathematical operators including $\{\times, +, -, \Lambda, \sqrt{\}}$ along with some random and constant numbers were used to develop relations.

4.2 Results of relation development and statistical analysis

In group A, as stated later, a relation with very appropriate accuracy was obtained. But, for group B, after several runs of GP, only one run with a 3-input relation was introduced; in group C, after several runs of GP, no relation including 3 considered primary variables was obtained. Thus, it can be concluded that RI_V and cylindrical compressive strength have a more determining effect on the tensile strength of SFRC. Tree relation resulted from executing GP code is presented in Fig. 4 for groups A and B. These relations are presented for groups A and B in Eq. (6) and Eq. (7), respectively, in the mathematical form with minimum simplification. By further simplification, Eq. 6 and 7 can be stated as Eq. 8 and 9.



Figure 4. Tree relations obtained from executing GP code; A) Tensile strength relation of SFRC for Group A; B) Tensile strength relation of SFRC for Group B

$$\frac{f_{sp}}{17.98} = \left(\frac{f_c}{107.02}\right) \times \left(0.42567 + \left(2 * \frac{RI_V}{3.1}\right)^{\left(\frac{f_c}{107.02}\right)} \times 0.39846\right)$$
(6)

$$\frac{f_{sp}}{17.98} = \left(\frac{f_c}{107.02}\right) \times \left(\left(\frac{RI_V}{3.1}\right) + 0.3784\right) + 0.064438 \times \frac{lf}{df \times 156}$$
(7)
+ 0.042019

$$f_{sp} = f_c \times (0.0715 + (0.645 \times RI_V)^{\left(\frac{f_c}{107}\right)} \times 0.067)$$
(8)

$$f_{sp} = f_c \times (0.541955 \times RI_V + 0.063573) + 0.007427 \times \frac{lf}{df} + 0.7555$$
(9)

In Tables 3 and 4, the proposed relations are compared with the relations presented so far in order to determine the splitting tensile strength of SFRC. In these tables, the superiority of the proposed relation, especially Eq. (8), is visible. In all the statistical indexes, Eq. (8) is superior to other relations (Table 3). It is worth noting that, in all previous studies, relations are obtained for all data, but their precision has not been controlled in the samples that are not used in relation development. High accuracy of Eq. (8) for the experimental data shows its absolute superiority in terms of predicting tensile strength of SFRC (Table 4). To compare the results of the proposed relation, the graph of the results predicted by Eq. (8) was drawn in contrast to the laboratory results (Fig. 5). It should be noted that the laboratory results are not generally without error. Factors such as human error, machine error, effect of environmental conditions on the test, manner of processing, and other factors that are not measured could have a significant impact on the results. Therefore, based on the laboratory results, especially for concrete, error-free relations can never be achieved and a limited amount of error is always left in relations. Mean absolute percentage error of equation 8 was approximately 14% and 16% for all the samples and for the experimental samples, respectively, which was the minimum error compared with other relations presented in the literature.



Figure 5. Graph of the results predicted by equation 8 versus laboratory results

MAPE ^{**}	$f_{sp_{predict}}/f_{sp_{exp}}$		SAE [*]	R^2	Equation	Ref.				
	variance	average			1					
14.2	0.187	1.0006	289	0.8477	$f_{sp} = f_c \times (0.0715 + (0.645 \times RI_V)^{\left(\frac{f_c}{107}\right)} \times 0.067)$	Eq. (8)				
14.2	0.186	0.9849	335	0.7844	$f_{sp} = \hat{f}_c \times (0.541955 \times RI_V + 0.063573) + 0.007427 \times \frac{lf}{df} + 0.7555$	Eq. (9)				
16.3	0.206	0.9502	443	0.6145	$f_{sp} = (0.614 + 0.4 \times RI_V^{1.029}) \times \sqrt{f_c}$	Muzmar [12]				
24.1	0.299	1.1171	502	0.5854	$f_{sp} = 0.104 \times f_c + 0.00795 \times RI_V$	Nadiya [11]				
16.2	0.197	0.8944	465	0.5089	$f_{sp} = (0.63 + 0.46 \times RI_V) \times \sqrt{f_c}$	Rjoub [10]				
15.9	0.190	0.9042	431	0.6622	$f_{sp} = 0.63\sqrt{f_{cu}} + 0.288 \times \sqrt{f_{cu}} \times RI_V$	Tomas [9]				
	* Sum of Absolute Errors = $\sum f_{sp_{predict}} - f_{sp_{exp}} $									
* *	* * Mean Absolute Percentage Error= mean $\left \frac{f_{sp_{predict}} - f_{sp_{exp}}}{f_{sp_{predict}}} \right \times 100$									

Table 3: Comparing the proposed relation for all the data

One of the indexes for investigating the accuracy of relations is to investigate the ratio of predicted to laboratory values. The average of this ratio for Relation 8 had the closest value to the unit, which represented the appropriate modeling of the problem by this relation. By comparing the results in Tables 3 and 4, it can be seen that other relations derived from the literature had more inappropriate responses to the experimental data, which may be due to having more limited statistical population. But, in the case of the proposed relations, because these data were not used in their relation development, poorer results were certainly expected for them.

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Table 4. Comparing the proposed relation for the experimental data (to samples)									
MAPE ^{**}	$\frac{f_{sp_{predict}}}{f_{sp_{exp}}}$ variance average		- SAE [*]	R^2	Equation	Ref.			
16.0	0.197	1.0242	72	0.8471	$f_{sp} = f_c \times (0.0715 + (0.645 \times RI_V)^{\left(\frac{f_c}{107}\right)} \times 0.067)$	Eq. (8)			
15.9	0.193	0.9498	84	0.7520	$f_{sp} = f_c \times (0.541955 \times RI_V + 0.063573) + 0.007427 \times \frac{lf}{df} + 0.7555$	Eq. (9)			
19.2	0.222	0.4879	114	0.4879	$f_{sp} = (0.614 + 0.4 \times RI_V^{1.029}) \times \sqrt{f_c}$	Muzmar [12]			
26.0	0.325	1.1202	115	0.4959	$f_{sp} = 0.104 \times f_c + 0.00795 \times RI_V$	Nadiya [11]			
20.4	0.211	0.8418	128	0.3207	$f_{sp} = (0.63 + 0.46 \times RI_V) \times \sqrt{f_c}$	Rjoub [10]			
19.6	0.206	0.8542	113	0.5522	$f_{sp} = 0.63\sqrt{f_{cu}} + 0.288 \times \sqrt{f_{cu}} \times RI_V$	Tomas [9]			

Table 4: Comparing the proposed relation for the experimental data (80 samples)

One of the benefits of the existence of relations with appropriate accuracy is better understanding of the materials behavior for optimum design with acceptable specifications. Relation 8 is the most comprehensive and accurate relation according to the presented materials, based on which graph of tensile strength versus cylinder compressive strength and fiber index was drawn (Fig. 6). According to Fig. 6, at low compressive strength, the effect of fiber index on tensile strength was almost equal and only the presence or absence of fiber was important. Gradually, the fiber effect was increased with an increase in compressive strength. In the class of concrete with medium strength, with an increase in RI_V , the effect of steel fiber on the increasing concrete tensile strength was reduced. It should be noted that the relation of increasing tensile strength of SFRC with RI_V in high-strength concrete (about MPa 90) became linear.



Figure 6. Graph of tensile strength of SFRC versus cylindrical compressive strength and fiber index

5. RESULTS

In this study, two equations were development to predict the tensile strength of SFRC using GP based on the results of 367 laboratory samples collected from the literature. These equations were compared with those presented in the literature. Also, based on these equations, effect of various parameters on the splitting tensile strength of SFRC was investigated. The results of this study are summarized as follows:

- Parametric investigations by GP showed that cylindrical compressive strength and amount of steel fibers index has a determining role on splitting tensile strength of SFRC.
- The proposed equations could determine the amount of splitting tensile strength of SFRC with higher accuracy than all the relations presented so far.
- In the concretes with low compressive strength, only the presence or absence of steel fibers could affect splitting tensile strength and fiber index was not important. In the medium-resistant concretes, with an increase in the steel fiber index, its effect on increased tensile strength of mixture was reduced. But, in the high-strength concretes, the relation between fiber index and tensile strength of SFRC was linear and direct.

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No.	Ref.	\hat{f}_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	f_{sp} (MPa)	No.	Ref.	\hat{f}_c (MPa)	$\frac{L_f}{D_f}$	V_f %	f_{sp} (MPa)
1	20	76.02	60	0.50	5.68	185	10	93.91	80	1.50	10.98
2	20	78.48	60	1.00	6.95	186	10	98.42	80	1.50	11.41
3	20	80.09	60	1.25	8.26	187	21	92.6	75	1.00	10.02
4	20	84.63	60	1.50	8.93	188	21	93.7	75	1.00	10.08
5	20	96.22	60	1.75	9.97	189	21	95.4	75	0.50	8.50
6	20	88.97	60	2.00	10.83	190	21	95.83	75	0.50	8.52
7	20	76.96	60	0.50	6.94	191	21	97.53	75	1.00	10.28
8	20	78.85	60	1.00	8.14	192	21	100.5	75	1.00	10.44
9	20	84.48	60	1.25	9.12	193	21	97.1	75	1.50	11.94
10	20	87.40	60	1.50	10.03	194	21	101.3	75	1.50	12.20
11	20	89.52	60	1.75	11.16	195	21	95.00	75	1.00	10.15
12	20	91.49	60	2.00	11.74	196	22	42.30	75	1.57	4.55
13	20	78.02	75	0.50	7.51	197	22	43.20	75	2.82	4.60
14	20	80.95	75	1.00	8.89	198	22	47.70	75	2.85	4.83
15	20	86.21	75	1.25	10.71	199	22	46.80	75	3.09	4.79
16	20	89.19	75	1.50	11.50	200	22	48.60	75	3.03	7.16
17	20	91.73	75	1.75	12.54	201	22	47.70	75	3.82	6.96
18	20	93.56	75	2.00	13.16	202	22	43.20	75	4.05	6.62
19	20	32.66	75	0.50	3.93	203	23	61.7	100	0.25	6.39
20	20	34.11	75	1.00	4.72	204	23	39.9	100	0.25	5.14
21	20	36.28	75	1.25	5.35	205	23	61.7	133	0.50	7.88
22	20	37.46	75	1.50	5.90	206	23	67.2	133	1.00	10.70
23	20	39.27	75	1.75	6.10	207	23	59.3	100	0.50	7.14
24	20	39.85	75	2.00	6.84	208	23	60.0	100	1.00	8.95
25	20	33.73	83	0.50	4.12	209	23	67.0	100	1.50	11.32
26	20	34.63	83	1.00	5.24	210	23	55.9	100	2.00	12.04
27	20	36.61	83	1.25	6.18	211	23	61.7	100	0.25	6.39
28	20	38.31	83	1.50	6.53	212	23	39.9	100	0.25	5.14
29	20	39.63	83	1.75	7.15	213	23	61.7	133	0.50	7.88
30	20	41.17	83	2.00	7.87	214	23	67.2	133	1.00	10.70
31	20	33.99	83	0.50	4.36	215	23	61.7	100	0.25	6.39
32	20	35.26	83	1.00	5.94	216	23	39.2	100	0.25	5.09
33	20	37.09	83	1.25	6.54	217	23	61.7	133	0.50	7.88
34	20	39.73	83	1.50	7.07	218	23	76.7	100	1.50	12.11
35	20	41.27	83	1.75	7.86	219	23	79.5	100	2.00	14.36
36	20	42.87	83	2.00	8.33	220	23	77.2	100	2.50	16.14
37	10	55.22	80	0.00	4.96	221	23	75.8	100	3.00	17.98

APPENDIX A - COLLECTED LABORATORY DATA

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No.	Ref.	f_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	<i>f_{sp}</i> (MPa)	No.	Ref.	f_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	<i>f_{sp}</i> (MPa)
38	10	63.71	80	0.00	5.11	222	23	42.3	133	0.50	6.52
39	10	71.06	80	0.00	5.71	223	23	41.4	100	1.00	7.43
40	10	80.87	80	0.00	6.34	224	23	55.7	133	0.50	7.48
41	10	91.85	80	0.00	6.62	225	23	42.3	133	0.50	6.52
42	10	59.76	80	0.00	5.18	226	23	71.9	133	1.00	11.07
43	10	65.65	80	0.00	5.26	227	23	67.0	100	1.50	11.32
44	10	74.69	80	0.00	5.91	228	24	40.192	100	3.10	5.71
45	10	84.84	80	0.00	6.74	229	24	40.193	100	3.10	5.71
46	10	94.78	80	0.00	6.77	230	24	40.193	100	3.10	5.71
47	10	65.25	80	0.00	5.28	231	24	40.193	100	3.10	5.71
48	10	70.21	80	0.00	5.47	232	24	40.193	70	3.10	5.33
49	10	79.51	80	0.00	6.12	233	24	40.193	70	3.10	5.33
50	10	89.27	80	0.00	6.58	234	24	40.193	70	3.10	5.33
51	10	98.92	80	0.00	6.93	235	24	40.193	70	3.10	5.33
52	10	48.74	80	0.00	3.8	236	24	39.71	70	3.10	6.18
53	10	58.81	80	0.00	4.01	237	24	39.71	70	3.10	6.18
54	10	67.31	80	0.00	4.34	238	24	39.71	70	3.10	6.18
55	10	74.92	80	0.00	4.63	239	24	39.71	70	3.10	6.18
56	10	80.77	80	0.00	4.81	240	24	39.71	70	3.10	6.18
57	10	51.22	80	0.00	3.95	241	24	39.71	70	3.10	6.18
58	10	60.85	80	0.00	4.1	242	24	39.71	70	3.10	6.18
59	10	69.39	80	0.00	4.42	243	24	39.71	70	3.10	6.18
60	10	80.11	80	0.00	4.75	244	24	39.71	70	3.10	6.18
61	10	86.48	80	0.00	4.98	245	25	42.49	75	0.00	4.56
62	10	57.87	80	0.00	4.08	246	25	41.90	75	0.00	4.53
63	10	64.08	80	0.00	4.25	247	25	41.90	75	0.00	4.53
64	10	75.31	80	0.00	4.63	248	25	42.49	75	0.38	5.40
65	10	84.42	80	0.00	4.82	249	25	39.70	75	0.50	5.49
66	10	93.64	80	0.00	4.15	250	25	41.42	75	1.00	6.70
67	10	60.14	80	0.50	5.86	251	25	40.11	75	0.38	5.24
68	10	74.63	80	0.50	6.52	252	25	42.67	75	0.50	5.69
69	10	82.67	80	0.50	7.25	253	25	40.47	75	1.00	6.62
70	10	91.31	80	0.50	8.12	254	25	40.85	75	0.75	6.11
71	10	95.65	80	0.50	8.43	255	25	40.47	75	1.25	7.17
72	10	63.07	80	0.50	6.10	256	25	40.11	75	0.50	5.51
73	10	78.66	80	0.50	6.79	257	25	41.42	75	1.00	6.70
74	10	88.30	80	0.50	7.36	258	26	32.17	63	0.00	2.73
75	10	93.94	80	0.50	8.81	259	26	36.33	63	0.50	3.33
76	10	97.06	80	0.50	8.95	260	26	41.55	63	1.00	4.50
77	10	66.23	80	0.50	6.32	261	26	38.40	63	1.50	3.51
78	10	79.42	80	0.50	6.97	262	26	35.31	63	2.00	3.23
79	10	90.79	80	0.50	7.92	263	26	33.37	63	2.50	2.88
80	10	94.13	80	0.50	8.92	264	26	31.66	63	3.00	2.82
81	10	99.94	80	0.50	9.02	265	26	42.56	43	1.00	4.72
82	10	52.63	80	0.50	4.68	266	26	41.90	56	1.00	4.25
83	10	61.72	80	0.50	4.97	267	26	40.60	100	1.00	4.40
84	10	74.67	80	0.50	5.51	268	9	29.80	55	0.00	3.93
85	10	82.89	80	0.50	6.15	269	9	30.50	55	0.50	4.37
86	10	91.07	80	0.50	6.26	270	9	31.20	55	1.00	4.87
87	10	54.71	80	0.50	4.94	271	9	32.30	55	1.50	5.43

No.	Ref.	f_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	<i>f_{sp}</i> (MPa)	No.	Ref.	f_c (MPa)	$\frac{L_f}{D_f}$	V_f %	f_{sp} (MPa)
88	10	67.91	80	0.50	5.94	272	9	56.00	55	0.00	5.19
89	10	79.31	80	0.50	6.28	273	9	57.00	55	0.50	5.81
90	10	89.76	80	0.50	6.63	274	9	57.80	55	1.00	6.49
91	10	93.62	80	0.50	6.91	275	9	59.40	55	1.50	7.33
92	10	58.11	80	0.50	5.34	276	9	72.40	55	0.00	5.76
93	10	69.37	80	0.50	6.04	277	9	73.60	55	0.50	6.48
94	10	81.82	80	0.50	6.42	278	9	74.80	55	1.00	7.20
95	10	91.13	80	0.50	6.81	279	9	77.00	55	1.50	7.98
96	10	95.66	80	0.50	7.07	280	27	22.34	0	0.00	1.96
97	10	62.51	80	0.75	7.13	281	27	26.19	128	0.40	4.50
98	10	76.71	80	0.75	7.80	282	27	28.57	128	0.70	4.60
99	10	85.62	80	0.75	8.62	283	27	29.73	128	1.00	4.74
100	10	92.98	80	0.75	9.64	284	27	24.62	96	0.40	3.60
101	10	97.03	80	0.75	9.83	285	27	25.24	96	0.70	3.88
102	10	65.21	80	0.75	7.32	286	27	25.38	96	2.00	4.07
103	10	80.42	80	0.75	7.95	287	27	23.79	64	0.40	3.12
104	10	90.17	80	0.75	8.90	288	27	24.76	63	0.70	3.64
105	10	94.97	80	0.75	9.75	289	27	25.17	64	1.00	4.01
106	10	98.11	80	0.75	9.92	290	28	84.94	60	1.02	10.01
107	10	67.37	80	0.75	7.80	291	28	90.09	60	1.52	11.78
108	10	82.66	80	0.75	8.37	292	28	97.89	60	1.53	10.86
109	10	91.89	80	0.75	9.20	293	28	107.02	60	1.02	7.21
110	10	95.62	80	0.75	9.95	294	29	60.72	63	0.00	4.32
111	10	99.98	80	0.75	10.31	295	29	61.89	63	0.50	5.88
112	10	54.01	80	0.75	6.05	296	29	66.54	63	0.75	6.08
113	10	62.89	80	0.75	6.48	297	29	29.88	63	0.50	3.83
114	10	76.63	80	0.75	7.08	298	30	42.30	75	0.00	4.73
115	10	84.62	80	0.75	7.91	299	30	43.10	75	0.50	5.61
116	10	92.82	80	0.75	8.08	300	30	44.80	75	1.00	6.87
117	10	55.84	80	0.75	6.41	301	30	44.10	75	1.50	6.71
118	10	69.42	80	0.75	6.41	302	30	42.10	75	2.00	6.60
119	10	81.61	80	0.75	7.68	303	31	33.60	0	0.00	2.25
120	10	90.31	80	0.75	8.20	304	31	35.20	150	0.30	2.81
121	10	94.28	80	0.75	8.57	305	31	37.04	150	0.60	3.27
122	10	58.77	80	0.75	6.63	306	31	37.70	150	0.90	3.81
123	10	71.77	80	0.75	7.38	307	31	38.40	150	0.12	4.12
124		84.03	80	0.75	7.77	308		32.48	0	0.00	2.20
125	10	92.71	80	0.75	8.37	309	31	33.85	150	0.30	2.46
126	10	96.00	80	0.75	8.84	310	31	34.44	150	0.60	3.35
127	10	64.73	80	1.00	8.12	311	31	34.95	150	0.90	3.75
128	10	78.91	80	1.00	8.93	312	31	35.25	150	0.12	3.95
129	10	88.01	80	1.00	9.69	313	32	36.10	50	0.50	4.51
130	10	94.67	80	1.00	10.9	314	32	44.82	50	1.00	4.79
131	10	99.21	80	1.00	11.4	315	32	36.03	50	1.50	4.58
132	10	67.31	80	1.00	8.41	316	32	44.08	50	2.00	4.60
133	10	81.14	80	1.00	9.15	317	32	24.47	50	0.50	2.93
134	10	92.93	80	1.00	10.0	318	32	25.50	50	1.00	3.49
135	10	96.62	80	1.00	11.10	319	32	16.80	50	1.50	2.82
136	10	99.98	80	1.00	11.50	320	32	18.75	50	0.50	1.97
137	10	69.71	80	1.00	8.81	321	32	30.40	50	1.00	3.49

No.	Ref.	\hat{f}_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	<i>f_{sp}</i> (MPa)	No.	Ref.	f_c (MPa)	$\frac{L_f}{D_f}$	$V_f\%$	<i>f_{sp}</i> (MPa)
138	10	84.31	80	1.00	9.42	322	32	24.08	50	1.50	2.96
139	10	93.76	80	1.00	10.72	323	32	12.50	50	0.50	1.93
140	10	96.20	80	1.00	11.63	324	32	8.23	50	1.00	1.30
141	10	100.12	80	1.00	11.92	325	33	44.90	0	0.00	3.60
142	10	55.20	80	1.00	6.83	326	33	46.50	97	1.00	5.18
143	10	64.73	80	1.00	7.44	327	33	48.32	97	1.50	5.55
144	10	78.98	80	1.00	8.12	328	33	52.16	97	2.00	6.11
145	10	87.03	80	1.00	8.97	329	33	31.60	0	0.00	2.51
146	10	93.91	80	1.00	9.20	330	33	33.70	97	1.00	3.95
147	10	57.88	80	1.00	7.23	331	33	37.12	97	2.00	4.96
148	10	73.07	80	1.00	7.92	332	33	43.68	97	3.00	5.67
149	10	82.6	80	1.00	8.45	333	33	32.80	49	2.00	3.75
150	10	92.61	80	1.00	9.57	334	33	37.20	49	4.00	4.21
151	10	95.31	80	1.00	9.92	335	33	43.20	49	6.00	5.49
152	10	59.62	80	1.00	7.89	336	33	35.30	81	1.00	4.24
153	10	72.78	80	1.00	8.30	337	33	36.24	104	1.00	4.54
154	10	86.21	80	1.00	8.97	338	33	37.76	156	1.00	5.13
155	10	93.24	80	1.00	9.64	339	33	33.76	75	1.00	3.45
156	10	97.21	80	1.00	10.08	340	33	33.68	97	1.00	3.68
157	10	66.81	80	1.50	9.22	341	33	35.68	104	1.00	3.81
158	10	80.82	80	1.50	9.98	342	33	29.30	78	1.00	3.35
159	10	91.00	80	1.50	10.73	343	33	29.44	47	1.00	3.05
160	10	96.73	80	1.50	11.68	344	33	32.50	97	1.00	3.56
161	10	100.18	80	1.50	12.27	345	33	42.10	97	1.00	5.15
162	10	69.08	80	1.50	9.71	346	33	43.90	97	1.00	5.38
163	10	82.29	80	1.50	10.74	347	33	43.90	97	1.00	5.38
164	10	94.1	80	1.50	11.2	348	33	46.80	97	1.00	5.61
165	10	96.35	80	1.50	12.04	349	33	35.04	97	2.00	4.61
166	10	100.20	80	1.50	12.63	350	33	36.60	97	2.00	5.10
167	10	71.23	80	1.50	9.85	351	33	36.64	97	2.00	5.07
168	10	85.12	80	1.50	10.84	352	33	36.64	97	2.00	5.07
169	10	94.7	80	1.50	11.9	353	34	57.60	0	0.00	5.55
170	10	98.7	80	1.50	12.7	354	34	40.20	Ő	0.00	4.20
171	10	101.30	80	1.50	13.08	355	34	26.40	0	0.00	3.30
172	10	56.77	80	1.50	7.68	356	34	59.76	150	0.70	6.30
173	10	66.82	80	1.50	7.94	357	34	42.00	150	0.70	5.25
174	10	80.11	80	1.50	8.72	358	34	32.40	150	0.70	4.05
175	10	88.18	80	1.50	4.98	359	35	32.75	0	0.00	2.80
176	10	94.22	80	1.50	10.30	360	35	35.39	60	1.00	4.80
177	10	59.12	80	1.50	8.21	361	11	18.10	0	0.00	1.98
178	10	74.24	80	1.50	9.00	362	11	23.70	22	0.40	2.91
179	10	84.63	80	1.50	9.64	363	11	29.70	22	0.40	3.76
180	10	93.11	80	1.50	10.36	364	11	34.80	22	1.20	4.82
180	10	96.72	80	1.50	10.30	365	36	17.27	0	0.00	3.00
181	10	61.31	80	1.50	8.36	366	36	22.02	50	1.00	5.20
182	10	73.66	80	1.50	9.24	367	36	25.22	50	1.00	5.80
185	10	87.11	80 80	1.50	9.88	507	50	23.22	50	1.00	5.00
10-	10	07.11	00	1.50	2.00						