

OPTIMUM SEISMIC DESIGN OF REINFORCED CONCRETE FRAMES USING IMPROVED METAHEURISTIC ALGORITHMS

S. Gholizadeh*,†, C. Gheyratmand and N. Razavi

Department of Civil Engineering, Urmia University, Urmia, Iran

ABSTRACT

The main objective of this study is to optimize reinforced concrete (RC) frames in the framework of performance-based design using metaheuristics. Three improved and efficient metaheuristics are employed in this work, namely, improved multi-verse (IMV), improved black hole (IBH) and modified newton metaheuristic algorithm (MNMA). These metaheuristic algorithms are applied for performance-based design optimization of 6- and 12-story planar RC frames. The seismic response of the structures is evaluated using pushover analysis during the optimization process. The obtained results show that the IBH outperforms the other algorithms.

Keywords: Structural optimization; performance-based design; improved metaheuristic; reinforced concrete frame.

Received: 2 March 2023; Accepted: 18 April 2023

1. INTRODUCTION

Population based metaheuristic algorithms are efficient tools in structural optimization and can be readily applied to problems with nonlinear constraints and discrete design variables. These kind of algorithms consist of two main phases: global search or exploration and local search or exploitation. One of the most important issues of metaheuristic algorithms is to establish an appropriate balance between these two main phases [1]. Consequently, it is reasonable to make some modifications to a metaheuristic algorithm to improve its performance in dealing with a specific class of optimization problems. Obviously, the optimization of reinforced concrete (RC) frames is a challenging class of structural optimization problems due to the complexity associated with reinforcement design and different cost components of the frame [2]. In the recent years, a number of studies have been conducted on the optimum design of the RC frames using metaheuristics [3-8].

*Corresponding author: Department of Civil Engineering, Urmia University, Urmia, P.O. box 165, Iran

†E-mail address: s.gholizadeh@urmia.ac.ir (S. Gholizadeh)

Performance-based design (PBD) [9] is a modern seismic design procedures for the design of structures subject to earthquake loading. In fact, PBD is a multilevel design approach in which several levels of structural performance are considered for corresponding hazard levels. In the framework of PBD structural seismic response is usually evaluated by performing structural nonlinear analysis. One of the major concerns of structural designers is to find cost-efficient structures having acceptable performance subject to earthquake. For this purpose, structural optimization methodologies have been developed in the last decades and structural performance-based optimal design (PBOD) becomes a topic of growing interest in the field of structural engineering. Due to the highly nonlinear and complex nature of the PBOD problem of structures, it is necessary to use global search algorithms such as metaheuristics to deal with this problem. Some researchers have recently studied the PBOD of RC frames using metaheuristics. Yazdani et al. [10] optimized RC frames in the context of probabilistic PBD using a modified discrete gravitational search metaheuristic algorithm. Gholizadeh and Aligholizadeh [11] proposed a chaotic enhanced colliding bodies optimization (CECBO) metaheuristic algorithm to solve deterministic and probabilistic PBOD problems of RC frames. Razmara Shooli et al. [12] dealt with PBOD problem of planar RC frames using a hybrid of genetic algorithm (GA) and particle swarm optimization (PSO). Razavi and Gholizadeh [13] used an improved black hole (IBH) metaheuristic algorithm to solve PBOD problem of RC frames.

This study focuses on the seismic optimization of planar RC frames in the framework of PBD using three improved metaheuristic algorithms including improved multi-verse (IMV) [4], improved black hole (IBH) [4] and modified newton metaheuristic algorithm (MNMA) [14]. Two illustrative design examples of 6-, and 12-story RC frames are presented. The nonlinear response of RC frames is evaluated using pushover analysis and to carry out the nonlinear structural analysis OpenSees [15] platform is used. The acceleration response spectra of the hazard levels specified by Standard No. 2800 [16] are considered as the target spectra. During the PBOD process, geometry, strength and strong column-weak beam constraints are checked according to ACI 318-08 code [17]. In addition, the PBD requirements are checked in accordance with ASCE 41-13 [18]. A total number of 30 independent optimization runs are carried out for statistical purposes and the performance of the IMV, IBH and MNMA metaheuristics are compared. The numerical results demonstrate the superiority of the IBH over the other algorithms.

2. PERFORMANCE-BASED OPTIMAL DESIGN OF RC FRAMES

According to the philosophy of PBD approach, the designed structures should meet a set of performance levels for a set of corresponding hazard levels. In this study, immediate occupancy (IO), life safety (LS) and collapse prevention (CP) performance levels is considered according to FEMA 356 and ASCE 41-13. Also, the acceleration response spectra of frequent, design and maximum considered earthquakes in accordance with Standard No. 2800 [16] are considered as the target spectra of seismic hazard levels with 50%, 10%, and 2% probability of exceedance in 50 years, respectively. The pushover analysis is conducted to evaluate the structural nonlinear responses. In which, the structure is pushed with a specific distribution of lateral loads, until the displacement of a specific point

of the structure reaches the target displacement. In this case, a set of preliminary checks including geometry, strength and strong column-weak beam constraints are considered according to ACI 318-08. Moreover, a set of design constraints are considered to assess the seismic performance of RC frames according to FEMA 356 and ASCE 41-13 as follows:

$$g_i^D = \frac{d^i}{d_{all}^i} - 1 \leq 0 ; i = \text{IO, LS, CP} \quad (1)$$

$$g_{i,j}^C = \frac{\theta_j^i}{\theta_{all}^{i,c}} - 1 \leq 0 ; i = \text{IO, LS, CP} ; j = 1, 2, \dots, nc \quad (2)$$

$$g_{i,j}^B = \frac{\theta_k^i}{\theta_{all}^{i,b}} - 1 \leq 0 ; i = \text{IO, LS, CP} ; k = 1, 2, \dots, nb \quad (3)$$

where d^i and d_{all}^i are the maximum inter-story drift and the allowable inter-story drift at i th performance levels, respectively; θ_j^i and θ_k^i are the maximum plastic hinge rotation of the j th column and k th beam, respectively; $\theta_{all}^{i,c}$ and $\theta_{all}^{i,b}$ are the allowable values of the plastic hinge rotation of the column and beam, respectively; and nc and nb are the number of columns and beams, respectively.

According to FEMA 356, $d_{all}^{IO} = 1\%$, $d_{all}^{LS} = 2\%$ and $d_{all}^{CP} = 4\%$. Also, the allowable plastic rotations of beams and columns at performance levels are determined using Tables 10-7 and 10-8 of ASCE 41-13.

In the optimization process of RC frames, cross sections of columns and beams are design variables and in this study are selected from Tables 1 and 2, respectively. the section databases of these tables are provided according to the specifications of ACI 318-08 [17].

Table 1: RC column section database

No.	Width (mm)	Depth (mm)	Number of D25 bars
1	400	400	4
2	400	400	6
:	:	:	:
48	900	900	22
49	900	900	24

Table 2: RC beam section database

No.	Width (mm)	Depth (mm)	Number of D22 bars	
			Positive	Negative
1	350	400	2	2
2	350	400	3	2
:	:	:	:	:
510	400	700	9	10
511	400	700	10	10

For the RC frames, the constructional cost (CC) associated with the concrete, steel, and formwork is defined as follows:

$$CC = \sum_{i=1}^{nb} (C_C b_{b,i} h_{b,i} + C_S A_{s.b,i} + C_F (b_{b,i} + 2h_{b,i})) L_i + \sum_{j=1}^{nc} (C_C b_{c,j} h_{c,j} + C_S A_{s.c,j} + 2C_F (b_{c,j} + h_{c,j})) H_j \quad (4)$$

where $b_{b,i}$, $h_{b,i}$, L_i , and $A_{s.b,i}$ are the i th beam width, depth, length, area of the steel reinforcement, respectively; $b_{c,j}$, $h_{c,j}$, H_j , and $A_{s.c,j}$ are the width, depth, height, and area of the steel reinforcement of the j th column, respectively; C_C , C_S , and C_F are the cost of the concrete, steel reinforcement, and formwork and the following values are considered for them: $C_C = 105 \text{ \$/m}^3$, $C_S = 0.9 \text{ \$/kg}$, $C_F = 92 \text{ \$/m}^2$.

The seismic PBOD problem of RC frames can be formulated as follows:

$$\text{Find: } X = \{x_1 \dots x_i \dots x_{nb+nc}\}^T \quad (5)$$

$$\text{To minimize: } CC(X) \quad (6)$$

$$\text{Sobjec to: } g_l(X) \leq 0, l = 1, 2, \dots, n \quad (7)$$

where X is vector of design variables; x_i is the design variable of i th element group; CC is the constructional cost of RC frames; and g_l is the l th design constraint; and n is the number of design constraints. In addition, the exterior penalty function method is used to handle the constraints during the optimization process.

3. IMPROVED METAHEURISTIC ALGORITHMS

The seismic PBOD problem of planar RC frames is solved using three improved metaheuristic algorithms in this work. The mathematical background of these algorithms are explained below.

3.1 Improved Multi-Verse

The multi-verse (MV) algorithm was developed based on the concepts of cosmology [20]. According to the basic concepts of multi-verse theory, there is more than one universe because more than one big bang was occurred. White holes were created due to inflation of universes and collision between them. In addition, black holes absorb everything in their vicinity and different objects were connected by wormholes. In the MV algorithm, each universe and each object are a candidate design and a design variable, respectively. In addition, for each universe, the inflation rate is proportional to the objective value of its corresponding candidate design. The universes with maximum and minimum inflation rates are considered as white hole and black hole, respectively. Objects can move between

different universes from the white holes towards the black holes and can randomly move through wormholes to the best universe [20]. The MV has been improved in [4] and the IMV algorithm is as follows:

In the framework of IMV, the following equation is used to exchange the objects through white/black hole tunnels:

$$x_{i,j} = \begin{cases} x_{k,j} & r_1 < NF(X_i) \\ x_{i,j} & r_1 \geq NF(X_i) \end{cases} \quad (8)$$

where $x_{i,j}$ is the j th design variable of i th candidate design; $x_{k,j}$ is the j th design variable of k th randomly selected candidate design; $NF(X_i)$ is normalized objective value of i th candidate design; and r_1 is a random number drawn from $[0,1]$.

The following mechanism is used to exchange the objects through wormholes:

$$x_{i,j} = \begin{cases} x_{b,j} + \left(\frac{t}{t_{max}}\right)^{\frac{1}{p}} (x_{l,j} + r_2(x_{u,j} - x_{l,j})) & r_3 < 0.5 \\ x_{b,j} - \left(\frac{t}{t_{max}}\right)^{\frac{1}{p}} (x_{l,j} + r_2(x_{u,j} - x_{l,j})) & r_3 \geq 0.5 \\ x_{i,j} & r_4 \geq WEP \end{cases} \quad (9)$$

$$WEP = \begin{cases} 0.2 + 0.8 \frac{t}{t_{max}} & t < 0.5t_{max} \\ 0.1 + 0.1 \frac{t}{t_{max}} & t \geq 0.5t_{max} \end{cases} \quad (10)$$

where $x_{b,j}$ is the j th variable of the best design; t is the current iteration; t_{max} is maximum number of iterations; p is an exploitation accuracy parameter; $x_{l,j}$ and $x_{u,j}$ are the lower and upper bounds of j th variable; r_2 , r_3 and r_4 are random numbers within the range of $[0,1]$; WEP is wormhole existence probability.

3.2 Improved Black Hole

The black hole (BH) algorithm was proposed in [21] based on the physical concept of black hole in space. Every black hole has a huge concentrated mass and if an object crosses its boundary, known as the event horizon, it cannot escape from the gravitational pull of black hole. The BH algorithm has been improved in [4] and the IBH algorithm is explained below. The BH has been improved in [4] and the IBH algorithm is as follows:

The black hole can absorb every objects around it using the following equations:

$$X^{t+1} = X^t + d_1 R_1 (X_b - X^t) + d_2 R_2 (X_s - X^t) \quad (11)$$

$$d_1 = a_1 + \omega \quad (12)$$

$$d_2 = a_2 + \omega \quad (13)$$

$$\omega = \left(1 - \frac{t}{t_{max}}\right)^{1.4} \quad (14)$$

where X^{t+1} is the position of an object at iteration $t + 1$; X_b is the position of black hole (the best solution obtained so far); X_s is the best position that each object could attain so far; R_1 and R_2 are vectors of random numbers within the range of $[0,1]$; $a_1 \in [2.2, 2.35]$ and $a_2 \in [0.1, 0.2, 0.3]$ are two coefficients; and ω is an inertia coefficient.

If an object crosses the event horizon, a new object replaces it. The radius of event horizon R_{EH} and distance between an object and black hole D are computed as follows:

$$R_{EH} = \frac{1}{\sum_{i=1}^{ps} \left(\frac{1}{F_i}\right)} \quad (15)$$

$$D = |F_B - F_i| \quad (16)$$

where R_{EH} is the radius of event horizon; F_B and F_i are the objective values of black hole and the i th object, respectively; and ps is the population size.

3.3 Modified Newton Metaheuristic Algorithm

Newton metaheuristic algorithm (NMA) [22] is a population based optimization algorithm designed on the basis of Newton's gradient-based iteration. The NMA requires the numerical approximations of the derivatives of the objective function to update the position of the population in the design space. Thus, in each iteration, the objective values of all particles are evaluated and the population is sorted in ascending order of the objective values. The NMA has been modified in [14] and the outline of MNMA is as follows:

In the MNMA, the position of i th particle is updated using the following equations.

$$X_i^{t+1} = X_i^t + \Delta X_i^t \quad (17)$$

$$\Delta X_i^t = \left(\frac{t}{t_{max}}\right) \cdot R_1^t \cdot \Gamma \cdot (X_{i-1}^t - X_{i+1}^t) + \left(1 - \frac{t}{t_{max}}\right) \cdot R_2^t \cdot (X_B - X_i^t) \quad (18)$$

$$\Gamma = \frac{\kappa^2 F(X_{i+1}^t) + (1 - 2\kappa)F(X_i^t) - (1 - \kappa)^2 F(X_{i-1}^t)}{2\kappa F(X_{i+1}^t) - 2F(X_i^t) + 2(1 - \kappa)F(X_{i-1}^t)} \quad (19)$$

$$\kappa = \frac{\|X_i^t - X_{i-1}^t\|}{\|X_{i+1}^t - X_{i-1}^t\|} \quad (20)$$

where R_1^t and R_2^t are vectors containing uniformly distributed random numbers between 0 and 1; and X_B is the best design found so far.

In the framework of MNMA, the NMA is implemented sequentially using the exterior penalty function method for handling the design constraint. In the first stage of MNMA, an initial population consisting of ps individuals is randomly generated in the design space, and

Eqs. (17) to (20) are used to perform an optimization process considering a small value for the penalty parameter. Therefore, the algorithm converges to an infeasible solution. In the next stage, a new population is generated in the neighborhood of the best solution found in the previous stage X_B . As a result, X_B is directly introduced into the new population and the rest of the population is randomly generated using the following equation:

$$X_i = \emptyset(X_B, \sigma X_B) \quad (21)$$

where \emptyset is a random normal distribution with the mean X_B and the standard deviation σX_B .

The penalty parameter r_p is updated for the new stage by a magnification factor γ . The most influential parameters on the convergence rate of the MNMA are σ and γ . The best values of these parameters are 0.1 and 10, respectively determined by sensitivity analysis. The optimization process is continued until one of the stopping conditions is satisfied.

4. NUMERICAL EXAMPLES

Two design examples of 6- and 12-story RC frames are illustrated. The compressive strength and modulus of elasticity of the concrete are supposed 28 MPa and 24.87 GPa, respectively. The reinforcement yield stress is assumed as 420 MPa and its modulus of elasticity as 200 GPa. The dead load of 29.420 KN/m and live load of 11.768 KN/m are applied to all beams. For RC sections, the Kent-Scott-Park model is utilized as the confined and unconfined concrete model. The parameters of the confined concrete are calculated according to the Mander stress-strain model [23].

4.1 Example 1: 6-story RC frame

The geometry and element groups of 6-story RC frame is shown in Fig. 1. The columns and beams are grouped in 6 and 3 design groups, respectively. For this frame, 30 independent optimization runs are performed using IMV, IBH and MNMA considering population size of $ps = 50$ and maximum number of iterations $t_{max} = 250$.

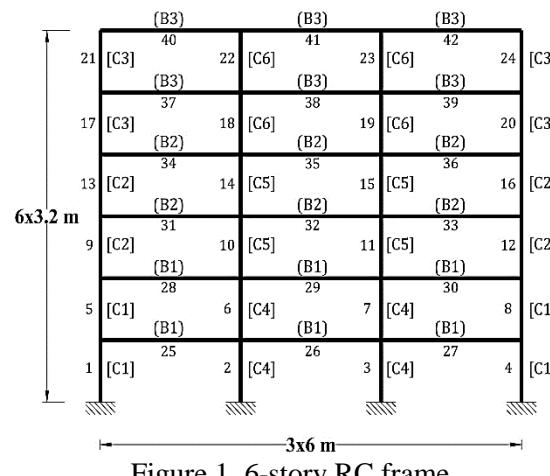


Figure 1. 6-story RC frame

The results of optimization are given in Table 3. The convergence curves of the best designs found by different algorithms are shown in Fig. 2.

Table 3: Optimization results for 6-story RC frame

Algorithm	Element		Dimensions (mm)		Reinforcement		Constructional Cost (\$)		
	Type	Group	Width	Depth	M ⁺	M ⁻	Best	Mean	SD
IMV	Column	C1	450	450	8-D25				
		C2	450	450	8-D25				
		C3	400	400	6-D25				
		C4	500	500	10-D25				
		C5	500	500	10-D25		35215	35496	468.42
		C6	400	400	8-D25				
	Beam	B1	350	650	2-D22	4-D22			
		B2	350	550	3-D22	3-D22			
		B3	350	500	2-D22	3-D22			
IBH	Column	C1	450	450	6-D25				
		C2	450	450	6-D25				
		C3	450	450	6-D25				
		C4	500	500	10-D25				
		C5	500	500	10-D25		34632	35293	331.67
		C6	400	400	8-D25				
	Beam	B1	350	650	2-D22	3-D22			
		B2	350	550	3-D22	3-D22			
		B3	350	400	3-D22	4-D22			
MNMA	Column	C1	450	450	6-D25				
		C2	450	450	6-D25				
		C3	450	450	6-D25				
		C4	550	550	8-D25				
		C5	550	550	8-D25		34873	35097	179.66
		C6	400	400	8-D25				
	Beam	B1	350	650	2-D22	3-D22			
		B2	350	550	2-D22	3-D22			
		B3	350	400	2-D22	4-D22			

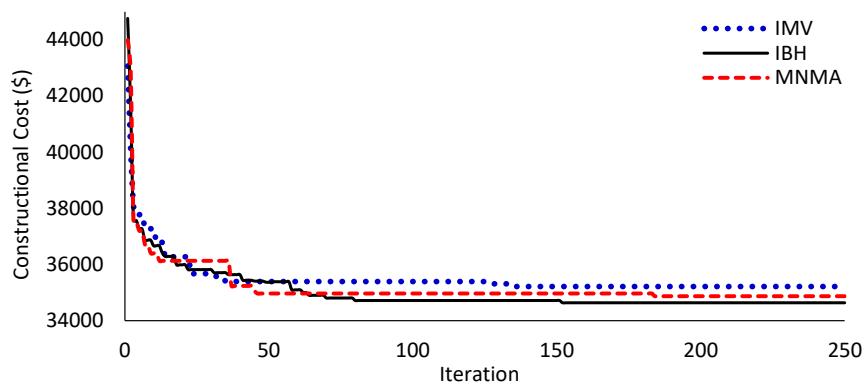


Figure 2. Convergence curves of the best designs for 6-story RC frame

It can be seen that the performance of IBH is better than the IVM and MNMA in terms of statistical results of optimization and convergence rate. Also, the second best algorithm is

MNMA. The inter-story drift ratios along the height of the 6-story RC frame for the best optimal designs found by different algorithms are shown in Fig. 3. The results show that the inter-story drift ratio constraint at LS performance level dominates the best optimal designs.

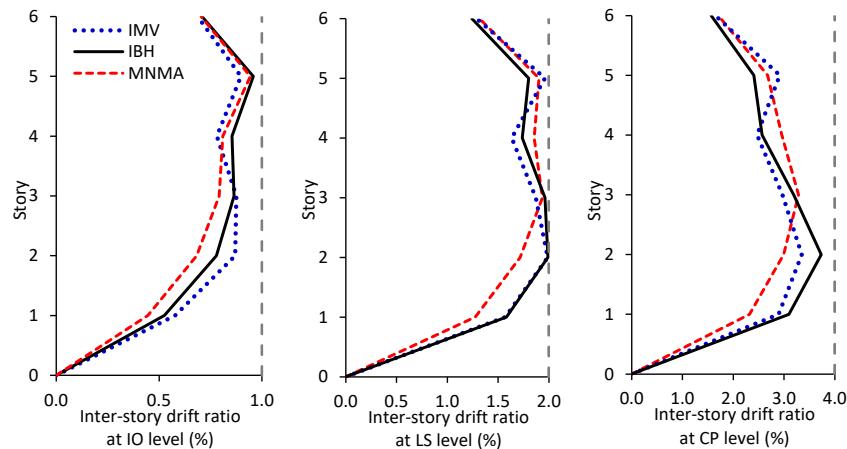


Figure 3. Inter-story drift ratio profiles for the best optimal designs of 6-story RC frame

4.2 Example 2: 12-story RC frame

The geometry and element groups of 12-story RC frame is shown in Fig. 4. The columns and beams are grouped in 12 and 6 design groups, respectively.

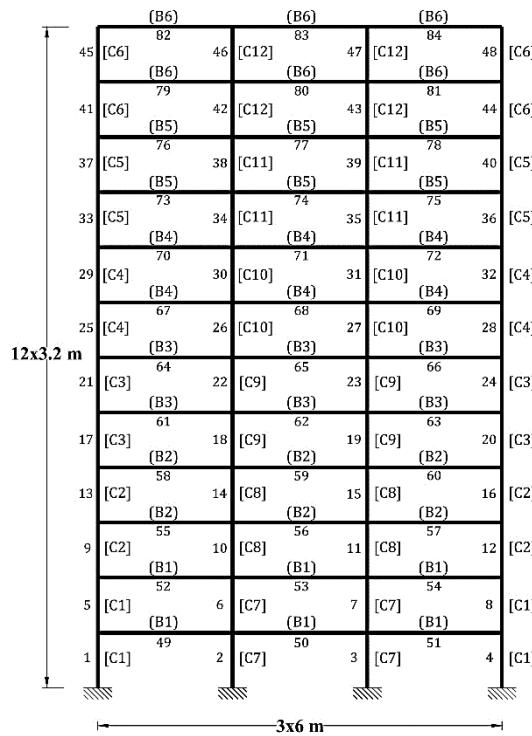


Figure 4. 12-story RC frame

Table 4: Optimization results for 12-story RC frame

Algorithm	Element		Dimensions (mm)		Reinforcement		Constructional Cost (\$)		
	Type	Group	Width	Depth	M ⁺	M ⁻	Best	Mean	SD
IMV	Column	C1	550	550	10-D25				
		C2	550	550	8-D25				
		C3	550	550	8-D25				
		C4	550	550	8-D25				
		C5	550	550	8-D25				
		C6	400	400	6-D25				
		C7	750	750	12-D25				
		C8	650	650	10-D25				
		C9	650	650	10-D25				
		C10	650	650	10-D25		81186	82071	751.29
		C11	450	450	10-D25				
		C12	450	450	8-D25				
IBH	Beam	B1	400	650	3-D22	3-D22			
		B2	400	650	3-D22	4-D22			
		B3	400	650	3-D22	3-D22			
		B4	400	550	2-D22	5-D22			
		B5	400	500	2-D22	3-D22			
		B6	350	450	2-D22	4-D22			
		C1	550	550	10-D25				
		C2	500	500	8-D25				
		C3	500	500	8-D25				
		C4	500	500	8-D25				
MNMA	Column	C5	500	500	8-D25				
		C6	400	400	6-D25				
		C7	750	750	12-D25				
		C8	650	650	10-D25				
		C9	650	650	10-D25				
		C10	650	650	10-D25		79098	80160	777.52
		C11	450	450	10-D25				
		C12	400	400	8-D25				
		B1	400	600	3-D22	4-D22			
		B2	400	700	3-D22	3-D22			
		B3	400	650	3-D22	3-D22			

In this example, 30 independent optimization runs are performed using IMV, IBH and MNMA considering population size of $ps = 50$ and maximum number of iterations $t_{max} = 500$. The results of optimization are given in Table 4. The convergence curves of the best designs found by different algorithms are shown in Fig. 5.

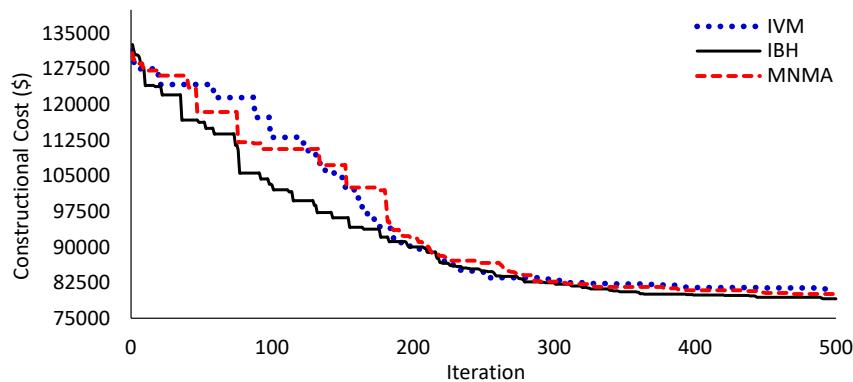


Figure 5. Convergence curves of the best designs for 12-story RC frame

It can be seen that the performance of IBH is better than the IVM and MNMA in terms of statistical results of optimization and convergence rate. Also, the second best algorithm is MNMA. The inter-story drift ratios along the height of the 12-story RC frame for the best optimal designs found by different algorithms are shown in Fig. 6. The results show that the inter-story drift ratio constraint at LS performance level dominates the best optimal designs.

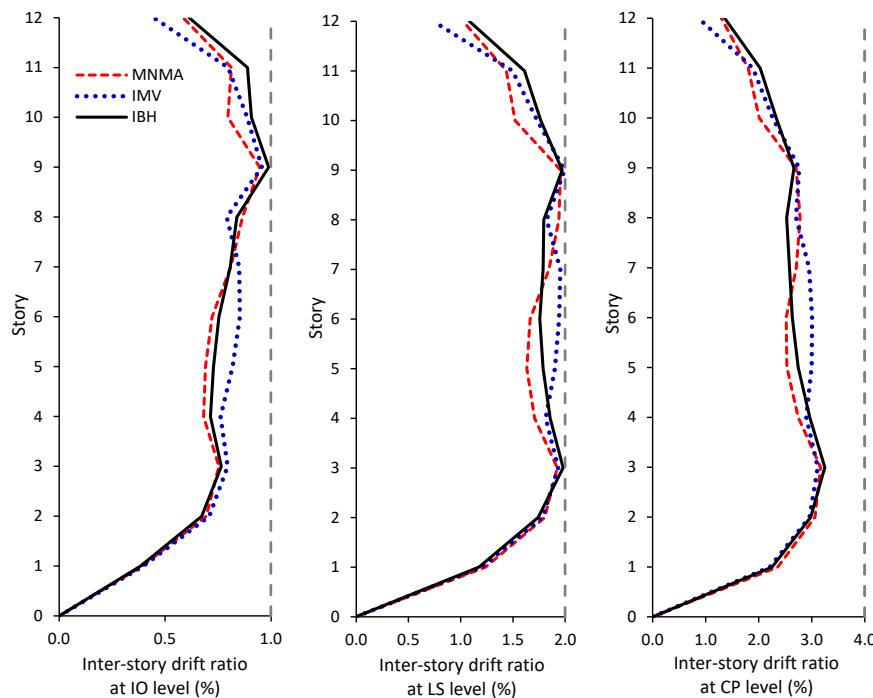


Figure 6. Inter-story drift ratio profiles for the best optimal designs of 12-story RC frame

5. CONCLUSIONS

Seismic optimization of planar RC frames is carried out in the context of PBD using three improved metaheuristic algorithms namely, IVM, IBH and MNMA. In the seismic design optimization process, preliminary checks are conducted according to ACI 318-08 and inter-story drift and plastic rotation checks are performed according to FEMA 356 and ASCE 41-13, respectively. In order to evaluate the nonlinear response of the structures during the optimization process, pushover analysis is performed. Furthermore, the constructional cost of RC frames is considered as the objective function to be minimized. Two design examples of 6-, and 12-story RC frames are presented and 30 independent optimization runs are achieved and the performance of the IMV, IBH and MNMA metaheuristics are statistically compared. For 6-story RC frame, the cost of the best solution found by IBH is 1.65% and 0.69% less than that of the IVM and MNMA, respectively. For the 12-story RC frame, the cost of the best solution found by IBH is 2.57% and 1.24% less than that of the IVM and MNMA, respectively. The obtained numerical results show that the performance of IBH is slightly better than both IMV and MNMA in terms of statistical results (including best, mean and standard deviation of optimal designs) and convergence rate.

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